Coalition Division as a Contested Garment:

The nucleolus, the discipline that sustains it, and the protection of small partners

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Abstract

A coalition government must hold its division of office period after period, against each member's temptation to defect. We study this as a repeated game of imperfect monitoring. Read as a claims problem, coalition division is a contested garment. The division that resolves the shortfall is the *nucleolus*, which protects small claims rather than scaling awards to them. A coalition's gain from defecting is its cooperative excess, so the monitoring sustains the garment division. What the monitoring sets, through the parties' demand game, is the over-claim. That fixes how far the division breaks from Gamson's proportional law. The break appears as a kink in the schedule of shares, its lower slope exactly one half. We take the prediction to the portfolio record of 308 governments: a non-parametric goodness-of-fit test rejects Gamson's proportional rule and does not reject the garment, pooled and in twelve of fourteen countries.

Keywords: coalition government, portfolio allocation, nucleolus, contested garment, bankruptcy problem, imperfect monitoring, repeated games, Gamson's law.

1 Introduction

A coalition government that involves compromise between different parties is not struck and then left. That compromise needs to be sustained period after period, against each member's standing temptation to defect from within: to withhold support for the common line and govern for itself, taking its own worth in place of its agreed share. That defection is not observed. Colleagues see only the government's standing, a noisy signal, and cannot tell a member pressing its own advantage from an adverse national tide; a government's fall is the eventual consequence of this. The coalitions literature has mostly studied the point at which governments form: the size of the coalition (Riker, 1962), how it divides the prize (Baron and Ferejohn, 1989; Morelli, 1999), and its stability in the sense that no party would rather deviate at the instant of the bargain. We ask what form a durable government, designed so that parties resist that temptation, might take. We study the division an ongoing coalition can sustain under imperfect monitoring, not the one struck at formation. The two need not coincide.

Read as a claims problem, coalition division is a *contested garment*. The contested garment is the Talmud's rule for an estate too small to meet the claims on it. It divides not in proportion but by protecting the smaller claims: each claimant is conceded what no other claims, and the contested remainder is split evenly (Aumann and Maschler, 1985). Coalition division is such a problem. The parties claim more office than there is to give, and the nucleolus (Schmeidler, 1969) provides its resolution. It rests on two foundations: an axiomatic one, and a bargained game based on players' demands (Serrano, 1995; Dagan, Serrano, and Volij, 1997).

The repeated game plays two roles. First, it sustains the garment. A coalition's gain from defecting is its cooperative excess, a standard identity in a new setting, and the monitoring deters that excess. The core of the bankruptcy game is non-empty, and holds both Gamson's proportional division and the nucleolus. So the monitoring sustains the garment but does not, on its own, single it out; the garment's pairwise consistency does that. Second, the monitoring sizes the over-claim, through the parties' demand game. That fixes how far the division breaks from Gamson's proportional rule: a half-slope kink, largest where monitoring is poorest, vanishing as it sharpens.

We model a repeated game in which a governing coalition of several parties holds a fixed office each period, sharing it by a division and governing on a common programme. Each member privately upholds the arrangement or deviates, taking its own worth in place of its agreed share; the deviation is not observed. What is public is a noisy signal of the coalition's standing, and discipline conditions on that signal alone: a bad enough draw triggers reversion to opposition, so punishment falls on the equilibrium path (Green and Porter, 1984). The per-period deterrence the monitoring affords is what any division must respect to be sustained, and the reliability of the monitoring fixes its size.

The paper delivers three results. First, coalition division is a contested garment. At two parties this is the classical two-claimant rule: each concedes what the other does not claim, and the contested remainder is split evenly, the symmetric nucleolus and the Nash bargain the two would strike (Rubinstein, 1982). Our result carries the rule to many parties, where the garment is the nucleolus of the bankruptcy game, which resolves the shortfall by protecting small claims rather than scaling awards to them. The nucleolus is not the proportional division and that fact organises the rest of the paper.

Second, monitoring sizes the break from proportionality. The monitoring sets the over-claim. The demand game pins it precisely at the value of opposition, what each party keeps by staying out of government, scaled by the chance a defector goes unseen. When monitoring is perfect the over-claim vanishes, and the garment is Gamson's proportional division. As monitoring worsens the over-claim grows, and the division breaks from Gamson toward the protective rule. The monitoring does not pick the division out of the core, which holds Gamson and the nucleolus alike. It sets only how far the garment sits from proportionality. Gamson is the perfect-monitoring corner; the contested garment is what imperfect monitoring leaves.

Third, the break from Gamson has a definite shape. Portfolio shares track seats for large partners. For small ones they bend up to a protective floor, a kink whose lower slope is exactly one half. That half-slope is what constrained equal awards, and only constrained equal awards, delivers. It separates the account from Gamson's law and from a reduced-form small-party bias.

From this we read the paper's central prediction: a half-slope kink in the portfolio schedule, its lower slope exactly one half. We take the prediction to the coalition portfolio record, where Gamson's law draws a straight line and the contested garment a kink. Across the 910 governing parties of 308 governments, a goodness-of-fit test rejects the proportional rule decisively and does not reject the garment. The pattern holds country by country: proportionality fails in twelve of fourteen democracies, the garment in one. The kink is the nucleolus at work. A coalition's temptation to defect is its excess, and the discipline can cover a temptation only up to the budget the monitoring affords, so the constraint that binds is the largest excess. The nucleolus is the division that makes that largest excess smallest: the most robust of those the monitoring can sustain, and the one the pairwise garment picks out.

The rest of the paper leads with the theorems. Section 2 places it alongside the relevant literature. Section 3 sets out the coalitional game, its programme and division, the signal, and the deterrence budget the monitoring affords. Section 4 asks which division that discipline sustains, and answers with an identity: the nucleolus. Section 5 reads the nucleolus as a claims problem and arrives at the contested garment of the Talmud; Section 6 draws out the half-slope kink. Sections 7–8 ground the claims in the deviation, and Section 9 pins the increment through a demand game. Section 10 takes the division to data, and Section 11 concludes.

2 Related literature

The subject of the paper is drawn from cooperative game theory. The nucleolus is due to Schmeidler (1969), and its identity with the Talmud's bankruptcy rule, the contested garment, is proved by Aumann and Maschler (1985), building on O'Neill (1982). The same division is also the outcome of a well-specified non-cooperative game: it is implemented for surplus-sharing problems by Serrano (1995), and for the bankruptcy rule by Dagan, Serrano, and Volij (1997), and it is reached in majority games by Montero (2006), the foundation closest to ours. We add a third reading: the nucleolus is the division a repeated imperfect-monitoring discipline sustains.

This interpretation places the paper in the Nash programme, the enterprise of grounding cooperative solution concepts in the equilibria of non-cooperative games (Nash, 1953; Serrano, 2005). For the nucleolus that programme has been carried out by design: the protocols just cited are engineered so their equilibrium is the nucleolus. We reach the same object from the other direction: the nucleolus is not implemented by a designer's mechanism but retrieved as the equilibrium of a game the coalition already plays, disciplined by what its members observe. The retrieval mirrors the two-player case, the Nash bargain of Rubinstein's (1982) alternating offers (Binmore, Rubinstein, and Wolinsky, 1986); the coalition plays the many-party counterpart. Our interpretation delivers a comparative static result: the division moves with the monitoring, so the same primitive that sustains the nucleolus also sets how far from Gamson's law it sits. Because the division emerges from the discipline imposed by the coalition and not from a designer's protocol, a built implementation gives no such result.

That discipline is a repeated game of imperfect public monitoring, in the tradition of Green and Porter (1984): cooperation held by the shadow of a punishment that falls, on the equilibrium path, on a bad draw of a noisy signal (Porter, 1983). The folk theorems supply the sustainable set, under perfect information (Friedman, 1971; Fudenberg and Maskin, 1986) and, through self-generation, under imperfect public monitoring (Abreu, Pearce, and Stacchetti, 1990; Fudenberg, Levine, and Maskin, 1994). The same discipline has lately been carried to political accountability (Acharya, Lipnowski, and Ramos, 2025), and the two-faction case, party discipline under the same monitoring, is the subject of Dewan (2026).

Coalition government is the application, with its own theory of formation and division. In the non-cooperative bargaining models minimal winning coalitions are predicted by Riker (1962); formation is modelled as an ordered schedule of proposals by Austen-Smith and Banks (1988), and as random recognition by Baron and Ferejohn (1989), which the evidence favours (Diermeier and Merlo, 2004). Demand bargaining is shown by Morelli (1999) to compete payoffs down to proportional shares, rationalising Gamson's (1961) regularity that portfolios track seats (Warwick and Druckman, 2006). A cooperative strand works from binding agreements: endogenous coalition structures are formed by Ray and Vohra (1997, 1999), and parties commit to a common platform in Levy (2004). These are theories of the bargain struck at formation; ours is of the bargain held afterward, and the two need not select the same division.

3 The model

Parties and government. A government is a winning coalition of n parties, its members $N = \{1, \dots, n\}$, drawn from a larger legislature and commanding a majority of its seats; we take the winning coalitions as given. Party $i \in N$ has an ideal point θ_i on a policy line and brings a share w_i of the coalition's seats, $\sum_{i \in N} w_i = 1$. The government is that coalition together with a *programme* $m \in [\min_i \theta_i, \max_i \theta_i]$ and a *division* x of the office it controls, the portfolios normalised to a unit pie: $x(N) = \sum_{i \in N} x_i = 1$, $x_i \geq 0$. Write $x(S) = \sum_{i \in S} x_i$ for any $S \subseteq N$. Time is discrete and infinite, and parties discount at $\delta \in (0, 1)$.

Actions and payoffs. Each period the standing programme and division are in place, and any coalition $S \subseteq N$ may *block*: its members jointly withhold support, forcing a re-bargaining in which S secures its worth $v(S)$ in place of the shares $x(S)$ it was receiving. Utility is transferable within the bloc, so S may divide $v(S)$ among its members in any way; a singleton block is one party pressing its own ideal or taking more than its share. Blocking costs the coalition $s > 0$ in standing, and it is not directly observed: it registers only through the noisy signal below, so a block is caught, and the government reverts, only with a probability the monitoring affords. What $v(S)$ is, and how re-bargaining grounds it, is the subject of Sections 5–8; for the logic it is enough that a block by S yields $v(S)$, freely divisible among its members. A member's stage payoff on the path is an office return drawn from the coalition's standing, plus its portfolio share, less a policy loss increasing in $|\theta_i - m|$.

Information. Actions are not observed. What is public is a noisy signal of the coalition's standing, and a shortfall cannot be told apart: a weak signal may be a deviation or bad luck. This is imperfect public monitoring in the sense of Green and Porter (1984): discipline conditions on the signal, and a bad enough draw triggers the punishment whether or not anyone in fact deviated, so the punishment falls on the path, on governments no one brought down. A deviation is caught only imperfectly. It raises the probability that the signal triggers reversion by a *detection reliability* $q \in [0, 1]$, higher the sharper the monitoring and zero when the signal is uninformative; a bloc S withdrawing together is caught with reliability q_S. We do not model how q arises here but treat it as a primitive of the monitoring process.

Histories, strategies, equilibrium. A *public history* at date t is the sequence $h^t = (y^0, \dots, y^{t-1})$ of realised signals, and $H = \bigcup_{t \geq 0} H^t$ the set of histories. A profile is a *disposition rule* $\alpha : H \to (m, x)$ giving the period's programme and division, together with a blocking rule for each coalition; both condition on the public signal alone, never on hidden actions. We study *stationary strong public perfect equilibria*: stationary public perfect equilibria (Fudenberg, Levine, and Maskin, 1994) that are also immune to coalitional blocks. Stationarity means the rules depend on the history only through the current phase, cooperation or reversion, and the standing

(m, x); the punishment is grim, a signal below a fixed threshold triggering permanent reversion to the opposition values r_i, the threshold's false-positive rate being the on-path collapse hazard β. The equilibrium is *strong* in that no coalition S, dividing $v(S)$ among its members by transfers, can leave each of them weakly better off than on the path. With transferable utility this fails, and the division holds against S, exactly when the bloc's one-shot gain $v(S) - x(S)$ does not exceed the discounted surplus $\delta\, q_S \sum_{i \in S}(W_i - R_i)$ it forfeits, with q_S the probability the block is caught.

Definition 1 (Deterrence budget)**.** κ is the discounted, detection-weighted punishment a deviation brings on: the fall in continuation value it induces, times the probability it is detected through the signal (made precise at (4)). It is the per-period deterrence the monitoring affords, rising with patience δ and with the informativeness of the signal, and falling with its noise.

Value functions and the deterrence budget. The budget κ is not a primitive; it is the continuation value a deviation risks losing, and the value functions make it explicit. On the path the government stands and a member i upholding it receives the stage payoff g_i, its office return and portfolio share net of the policy loss. The signal is imperfect, so even under universal advocacy a bad draw triggers reversion with probability β; otherwise the government stands to the next period. Writing W_i for the value of the standing government to i, and R_i for the reversion it collapses to with per-period outside value r_i,

$$W_i = g_i + \delta\big[(1-\beta)W_i + \beta R_i\big], \qquad R_i = r_i + \delta R_i, \tag{1}$$

so the surplus cooperation preserves is

$$W_i - R_i = \frac{g_i - r_i}{1 - \delta(1-\beta)}. \tag{2}$$

Now let a coalition S weigh abandoning the division. Withdrawing secures $v(S)$ this period in place of $x(S)$, a one-shot gain equal to the excess $e(S, x) = v(S) - x(S)$. But withdrawal is a deviation, and it raises the chance the signal triggers reversion by the detection reliability q for a lone defector, and q_S for a bloc, destroying the surplus (2). Writing the detection as q, the coalition holds rather than walks when

$$e(S, x) \;\le\; \delta\, q \sum_{i \in S}(W_i - R_i) \;\equiv\; \kappa. \tag{3}$$

The right-hand side is the deterrence budget of Definition 1, now in the primitives:

$$\kappa = \delta\, q\, \frac{\sum_{i \in S}(g_i - r_i)}{1 - \delta(1-\beta)}, \tag{4}$$

the discounted (δ), detection-weighted (q) surplus that reversion destroys. It rises with patience, with the detection reliability, and with the surplus of governing over opposition; it falls with the noise β. Taking the surplus and detection common across coalitions, the symmetric benchmark, gives the single budget of Definition 1; in general the constraint is coalition-specific, $e(S, x) \leq \kappa_S$, with κ_S the coalition-specific budget, and the sustainable set the corresponding core.[1] The detection reliability q that weights the budget is where the monitoring enters; how it arises from what a coalition can observe we do not model here; the logic of Section 4 reads (3) across every coalition. We assume throughout that parties are patient enough, and the signal informative enough, that a stationary public equilibrium with this reversion structure exists and its sustainable set is non-empty; for repeated games under imperfect public monitoring such existence is standard (Green and Porter, 1984; Fudenberg, Levine, and Maskin, 1994). The budget turns on the detection reliability q; the logic needs only that a deviation by S is deterred when its excess does not exceed κ.

4 The division the discipline sustains

We can now ask which division the discipline sustains, and the answer runs through a single identity. For the division results we adopt the symmetric benchmark of Section 3.

Assumption 1 (Symmetric monitoring)**.** The deterrence budget is common across the coalitions that bind, $\kappa_S \equiv \kappa$: the surplus $\sum_{i \in S}(g_i - r_i)$ and the detection weighting do not vary with S.

Under Assumption 1 a single κ governs the sustainable set. When the budget varies with S, that set is a weighted strong core and its centre the corresponding weighted nucleolus, which coincides with Schmeidler's nucleolus under Assumption 1. The *excess* of a coalition S at a division x is

$$e(S, x) = v(S) - x(S), \tag{5}$$

the amount S gains by taking its own worth $v(S)$ in place of its shares $x(S)$; by the model it is exactly the one-shot gain from defecting. The excess is the standard object of cooperative game theory, the amount by which S can improve on x and the quantity that defines the nucleolus; it drives the non-cooperative selection of the nucleolus elsewhere (Montero, 2006). What is new here is its repeated-game reading: under imperfect monitoring the excess is the per-period temptation the discipline must deter, and that is what ties the sustainable set to the cooperative cores.

Proposition 1 (Temptation is excess; sustainability is the strong core)**.** *The one-shot gain to a coalition S from abandoning the division x is its excess $e(S, x)$. Consequently, under*

[1] The stage payoff g_i is the return to governing plus the portfolio share x_i, so $\sum_{i \in S}(g_i - r_i)$, and hence κ_S, depends on the division through $x(S)$. This does not change the character of the constraint. The dependence is linear, so $e(S, x) = v(S) - x(S) \leq \kappa_S(x)$ stays linear in x, and the sustainable set is still a strong core on a rescaled excess. It is second-order, moreover, when the stake in cooperation is the value of holding office over opposition rather than the marginal portfolio, which is the reading we take.

Assumption 1, a stationary division x is sustainable in the reversion equilibrium of Section 3 if and only if

$$e(S,x) \leq \kappa \quad \textit{for every } S \subsetneq N, \tag{6}$$

where κ is the per-period deterrence the monitoring affords (increasing in patience and in detection power, decreasing in noise). The set of sustainable divisions is thus the strong κ-core $C_\kappa = \{x : \max_S e(S,x) \leq \kappa\}$.

Proof. By blocking, S secures $v(S)$ in place of $x(S)$; with transferable utility its members can share this take in any way, so some division of it leaves every member weakly better off than on the path if and only if the bloc's total one-shot gain $v(S) - x(S) = e(S,x)$ exceeds the total discounted surplus its members forfeit under reversion, $\delta\, q_S \sum_{i \in S}(W_i - R_i) \equiv \kappa$, the incentive constraint (3). Hence the strong equilibrium sustains x against S exactly when $e(S,x) \leq \kappa$; the member-level participation constraints are subsumed, since with transfers a profitable block requires only that the bloc's total gain beat its total forfeited surplus. Applying this to every coalition, the binding constraint being the one of greatest excess, gives (6) and the strong κ-core C_κ. □

That the discipline deters every coalition, not only every player, makes the equilibrium strong in the sense of Aumann (1959): with transfers, immunity to a block by every S is the core condition $e(S,x) \leq \kappa_S$. This is more than coalition-proofness (Bernheim, Peleg, and Whinston, 1987), which asks only that no self-enforcing sub-block improve. Here the distinction does not bind: a caught block collapses the government for all its members, so a block need not be self-enforcing to be deterred. A bargain that must withstand coalitional threats is also the object of Ray and Vohra (2024), albeit attained via a different route. The core itself has a non-cooperative foundation in the coalitional bargaining of Perry and Reny (1994); ours is a repeated-game one: the block is deterred by monitoring rather than by bargaining.

What the strong core contains depends on the game. Write $\varepsilon^* = \min_{x:\, x(N)=1} \max_S e(S,x)$ for the least worst-excess, and note $\kappa \geq 0$: a punishment cannot destroy more than the surplus at stake. When the static core is empty, $\varepsilon^* > 0$, no division escapes every coalition in one shot; the winner-take-all majority game, $v(S) = 1$ for each winning S, is of this kind, and there the discipline manufactures a non-empty sustainable set for $\kappa \geq \varepsilon^*$ whose robust centre is the nucleolus. The bankruptcy game we analyse in Section 8 is not of this kind: its core is non-empty, $\varepsilon^* \leq 0$, and holds both Gamson's proportional division and the nucleolus, so a non-negative budget sustains them together and cannot select one.

Proposition 2 (The nucleolus is the sustainable garment division)**.** *For the bankruptcy game whose claims satisfy $c_i = w_i + a \geq w_i$, the strong core is non-empty and contains the proportional division. The nucleolus ν of (N,v) (Schmeidler, 1969) lies in it and has the least worst-excess. It is thus a sustainable division, the most robust of them; the monitoring sustains it but does not, through a scalar budget, single it out from the rest of the core.*

Proof. Since $c_i = w_i + a \geq w_i$, for every S the reversion value $v(S) = \max\{0, 1 - \sum_{j \notin S} c_j\} \leq 1 - \sum_{j \notin S} w_j = w(S)$, so at the proportional division $e(S, w) = v(S) - w(S) \leq 0$ for all S. Hence w lies in the strong 0-core, which is non-empty, and $\varepsilon^* \leq 0$. The nucleolus is the lexicographic minimiser of the excesses (Schmeidler, 1969), so it attains ε^* and has the least worst-excess. Both w and ν have every excess $\leq 0 \leq \kappa$ for any $\kappa \geq 0$, so both are sustained, and a scalar budget does not separate them. □

The monitoring does not select the nucleolus over Gamson; both lie in the core and both are sustained for every $\kappa \geq 0$. The nucleolus is singled out among the sustainable divisions by the garment, an axiom (Schmeidler, 1969) and a bargaining protocol (Aumann and Maschler, 1985; Proposition 5). What the monitoring sets, through the demand game, is the over-claim the parties bring, and with it how far the garment division breaks from Gamson.

The over-claim is set by the monitoring. The demand game of Section 9 pins it at $a = (1-q)L$, falling to zero as the signal sharpens. At $a = 0$ the claims are the seats, there is no shortfall, and the garment is Gamson's proportional division; as monitoring worsens a rises, the claims over-run the estate, and the garment breaks from Gamson toward the protective rule, by the half-slope of Section 6. Gamson is thus the perfect-monitoring corner and the contested garment the noisy interior, and the size of the break is set by the reliability of the monitoring.

This locates the monitoring's impact precisely: not in selecting a division from the core, but in sizing the over-claim the garment resolves. Everything now turns on $v(S)$: what a coalition can command by defecting, and how large the claims that generate it grow.

5 Coalition division is a contested garment

Parties do not defect for nothing, nor, credibly, for everything. Each brings a claim on office. When the claims of the parties exceed the office there is to give, the division is a *claims problem*: an estate E (the portfolios) and claims $c = (c_1, \ldots, c_n)$ with $\sum_i c_i > E$. Its two-claimant kernel is the contested garment of the Mishnah, and its coalitional form is the bankruptcy game

$$v(S) = \max\big\{0,\ E - \textstyle\sum_{j \notin S} c_j\big\}, \tag{7}$$

what S can guarantee once the others are conceded their claims.

Aumann and Maschler (1985) showed that the nucleolus of this game is the Talmud's own division rule: constrained equal awards on the half-claims below the pivot $E = \frac{1}{2}\sum_i c_i$, and constrained equal losses above it. That rule is not the proportional rule. The Talmud divides a contested estate by protecting the smaller claims, not by scaling awards to claims. Non-cooperative strategic justifications for the family of rules it belongs to are given by Moreno-Ternero, Tsay, and Yeh (2022). Combined with Proposition 2:

Proposition 3 (The garment representation)**.** *When coalition division is the claims problem $(E;c)$, the sustainable garment division is the Talmud rule on $(E;c)$. It coincides with the proportional division only in the degenerate case of no shortfall, $\sum_i c_i = E$; whenever claims exceed the estate it breaks from proportionality, protecting small claims.*

Proof. By Proposition 2 the sustainable garment division is the nucleolus of (N,v). For the bankruptcy game $v(S) = \max\{0, E - \sum_{j \notin S} c_j\}$, the nucleolus is the Talmud rule on $(E;c)$: constrained equal awards on the half-claims below the pivot $E = \frac{1}{2}\sum_i c_i$, constrained equal losses above it. When $\sum_i c_i = E$ there is no shortfall and the rule awards each claimant its claim; whenever $\sum_i c_i > E$ it caps the awards on the smaller claims, protecting them, and so breaks from proportionality. □

How far the division breaks from proportionality grows with the over-claim, the gap $\sum_i c_i - E$. When the gap is zero the rule is proportional. As it widens the rule protects small claims ever more, reaching equal division when every party claims the whole estate. The precise break depends on the whole claims vector, not on the ratio $\sum_i c_i/E$ alone: two problems can share a ratio yet divide differently. In the structure this paper brings, $c_i = w_i + a$, the seats are fixed and the single increment a governs the break. The proportional benchmark of Gamson is the no-shortfall corner of a wider object, not the object itself.

6 The claims rule and the shape of the division

The garment names the division; what shape it takes, as a schedule in the seat share, is the next question. To read the shape we must say what a party claims. The natural rule is that a party claims its seats plus an increment: the value it names for opposition, the standing it gives up by governing, which it asks over and above its proportional share, and which is roughly common across parties and independent of size. With w_i the within-coalition seat share of Section 3 ($\sum_i w_i = 1$),

$$c_i = w_i + a, \qquad a \geq 0. \tag{8}$$

The estate is $E = 1$; the over-claim is $\sum_i c_i - E = na$. For modest aspirations ($na < 1$) the problem sits in the constrained-equal-losses branch, where each party's award is

$$x_i = \max\Big\{\tfrac{c_i}{2},\ c_i - \lambda\Big\}, \qquad \sum_i \min\Big\{\tfrac{c_i}{2}, \lambda\Big\} = na, \tag{9}$$

with λ the common loss. This is a continuous, piecewise-linear function of seats with a single kink at $w^* = 2\lambda - a$.

Proposition 4 (The kinked share function)**.** *Under $c_i = w_i + a$ the portfolio share is*

$$x_i = \begin{cases} \dfrac{a}{2} + \dfrac{1}{2}\,w_i, & w_i < w^* \quad (\textit{small parties}), \\[2ex] w_i - (\lambda - a), & w_i \geq w^* \quad (\textit{large parties}), \end{cases} \tag{10}$$

continuous at (w^*, λ), with $w^* = 2\lambda - a$. Small parties keep half their claim; the smallest, those with $w_i < a$, are lifted above their seats, while the rest below the kink are held below proportional but above a pure loss; large parties are shifted down to finance the floor. As $a \to 0$ the kink vanishes and $x_i = w_i$: Gamson's law is the no-aspiration limit.

Proof. In the constrained-equal-losses branch the loss borne by i is $\min\{c_i/2, \lambda\}$. A party with $c_i/2 \leq \lambda$, that is $w_i \leq 2\lambda - a$, bears the capped loss $c_i/2$ and keeps $x_i = c_i - c_i/2 = (w_i + a)/2$; a party with $c_i/2 > \lambda$ bears λ and keeps $x_i = c_i - \lambda = w_i + a - \lambda$. Setting $w^* = 2\lambda - a$ gives the two pieces; at $w_i = w^*$ both equal λ, so the function is continuous. When $a = 0$ the over-claim is zero, $\lambda = 0$, and $x_i = w_i$. □

The content of Proposition 4 is the slope. Below the kink the share rises with seats at a slope of one half: these parties sit at exactly half their claim, the Talmud rule's half-claim floor. Small parties are neither ignored nor rewarded in proportion; they are protected, each keeping half of what it asks. Above the kink the slope is one, as Gamson would have it, but displaced downward by the cost of that protection.

An example. Take a three-party government with twenty cabinet posts and within-coalition seat shares of twelve, five, and three. Proportionality, Gamson's law, hands out the posts twelve, five, three. Suppose instead each party claims its seats plus a floor of five posts, the minimum it wants to make joining worthwhile: claims of seventeen, ten, and eight, which together demand thirty-five posts of the twenty there are. The claims exceed the estate; it is a bankruptcy, a contested garment. The Talmud rule divides it not in proportion but by protecting the smaller claims. Because the estate exceeds half the claims, the parties share the shortfall by equal losses, save that none is cut below half of what it asked. The small party's half-claim is four posts, and that is its floor: it keeps four, one more than its three seats. The largest party absorbs the difference, falling from twelve to eleven; the middle party is unchanged at five. The division is eleven, five, four. The small party is over-rewarded relative to its seats, and the two protected parties sit at exactly half their claims, the half-claim floor that identifies the garment. It is the two-claimant garment scaled up: as the claimant to half a cloth is guaranteed half of it, the small party is guaranteed half of what it asks, and the dispute over the surplus is settled among the larger claimants. Figure 1 sets the two divisions of the pie side by side, and Figure 2 plots the schedule across seat sizes: the kink, with its half-slope below and unit slope above.

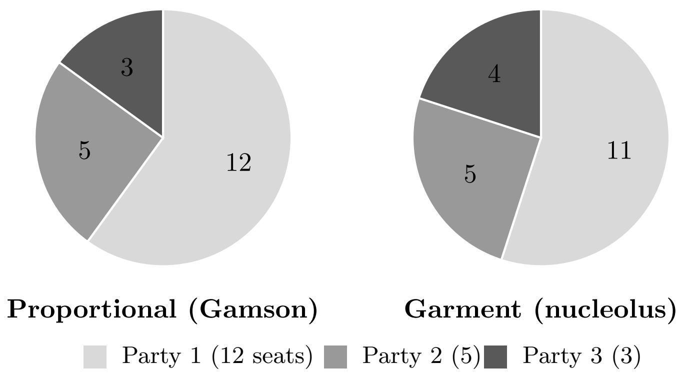

Figure 1: The divided pie. A three-party government of twenty posts, seats 12/5/3, claims 17/10/8. Proportionality hands out 12/5/3; the contested garment divides 11/5/4, seating the two protected parties at exactly half their claim (10/2 = 5, 8/2 = 4) and leaving the largest party to absorb the shortfall. The small party is lifted above its seats, from three posts to four.

How the claims become the division. The figures apply the bargaining rule Aumann and Maschler read out of the garment. When two parties claim c_1, c_2 from an amount t they share, each concedes what the other's claim leaves uncontested and the rest is split evenly, so party i receives

$$\tfrac{1}{2}\big(t - \max\{0, t - c_i\}\big) + \tfrac{1}{2}\max\{0, t - c_j\}. \tag{11}$$

A claimant to a whole cloth and a claimant to half of it take three-quarters and one-quarter, not two-thirds and one-third. For more than two parties the Talmud rule is the unique division under which every pair, sharing what the two are jointly awarded, divides it by (11); and it fixes our figures. The three parties give up $35 - 20 = 15$ posts between them. The loss falls equally, capped at half of each claim, so the two smaller parties lose their half-claims, five and four, and the largest absorbs the remaining six: the awards are $11, 5, 4$. The division is a garment pair by pair. Parties one and two are awarded sixteen together; the second concedes the six its claim of ten leaves uncontested, the first concedes nothing, since it claims more than the sixteen, the contested ten splits evenly, and they take $6 + 5 = 11$ and $0 + 5 = 5$. Parties one and three split fifteen into eleven and four the same way, and parties two and three split nine into five and four. Every pair divides as a garment, and that is what makes $11, 5, 4$ the nucleolus.

The nucleolus is reached from every side. As an axiom it is the lexicographic minimiser of the sorted excesses (Schmeidler, 1969); as a bargaining protocol it is Aumann and Maschler's concede-and-divide; and it is the equilibrium of a well-specified bargaining game, for Serrano (1995) implements the nucleolus for surplus-sharing problems as a subgame-perfect equilibrium, and Dagan, Serrano, and Volij (1997) the bankruptcy rule; our (N, v) is a bankruptcy game, so both deliver the garment division. The demand game of Section 9 is the coalition-government reading of that equilibrium, with Gamson as its frictionless limit.

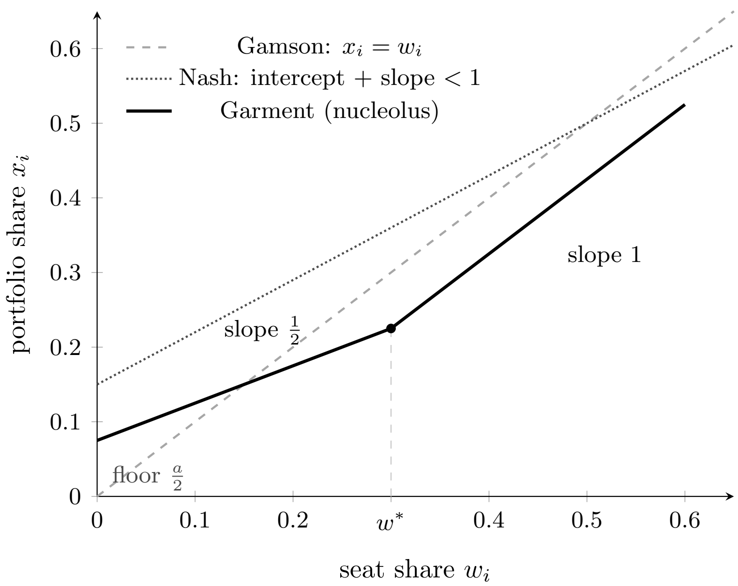

Figure 2: The half-slope kink. Portfolio share against seat share, for the three accounts. Gamson's law is the 45-degree line, shares proportional to seats. The reduced-form small-party bias is the Nash line, a positive intercept and a constant slope below one. The garment rule is the kink: small partners keep half of what they claim, so their share rises with seats at slope one-half from a floor of $a/2$, while large partners are shifted below proportional at slope one. The lower slope of exactly one-half is what constrained equal awards, and only constrained equal awards, delivers, and it separates the garment from Gamson's law and the reduced-form small-party bias alike.

We can, moreover, derive a bargaining game for it from the garment's own pairwise logic. Where bilateral bargaining over marginal contributions retrieves the Shapley value (Gul, 1989), bilateral re-settlement by the two-party garment retrieves the nucleolus.[2]

Proposition 5 (A bargaining game that derives the garment)**.** *Suppose any two partners may re-open the division of their combined award and settle it by the two-party contested garment* (11)*. A division is invariant under every such bilateral re-settlement if and only if it is the Talmud rule, the nucleolus. The garment is thus the fixed point of bilateral re-bargaining built from the two-party garment alone, and the two-party garment is itself the outcome of a bilateral concession game, so the coalition game rests on no cooperative primitive, and bilateral re-bargaining converges to it.*

Proof. The rest point is the pairwise-garment division of (11), pinned by the consistency that made $11, 5, 4$ the nucleolus above, and re-bargaining converges to it. In the regime of modest aspirations, $na < 1$, the garment sits in the constrained-equal-losses branch; write $\ell_i = c_i - x_i$ for i's loss, with $\sum_i \ell_i = na$ fixed and $\ell_i \leq c_i/2$. There the two-party garment on a pair is exactly *capped equalisation* of their losses: it moves ℓ_i, ℓ_j toward equality, subject to the caps,

[2]The proof uses a Lyapunov, or potential, argument: it exhibits a quantity that falls at every step of the re-bargaining and is bounded below, so the process must converge. The device is standard in dynamical systems and in consensus dynamics, but uncommon in cooperative game theory, where solutions are usually characterised statically rather than reached by a dynamic. Here that quantity is the sum of squared losses, dually the sum of squared awards, which each bilateral re-settlement lowers until the pair is capped-equal.

holding their sum fixed. From (11), when both claims exceed the pair's joint award the contested amount splits evenly, equalising the losses, and the half-claim cap binds when a claim is small. Each such move weakly lowers $\sum_i \ell_i^2$, strictly unless the pair is already capped-equal, and the sum of squares is bounded below; so re-bargaining, sweeping every pair, converges. That the rest point is unique, the constrained-equal-losses allocation and so the Talmud rule, is Aumann and Maschler's consistency theorem; the Lyapunov argument, whose function is $\sum_i \ell_i^2$, adds convergence to it. The sum of squares is a convergence device within this branch, not a global objective; minimising squared excesses over all divisions would select a different rule, and it is consistency, not the potential, that makes the rest point the nucleolus. The other regime is the mirror image. When aspirations are large, $na > 1$, the estate falls short of half the claims and the garment sits in the constrained-equal-awards branch, where each award is capped at half its claim, $x_i = \min\{c_i/2, \mu\}$ with $\sum_i x_i = 1$. There the two-party garment on a pair is capped equalisation of their awards: for a small joint total it splits equally, and it caps each at its half-claim, holding the pair's total award fixed. Each such move weakly lowers $\sum_i x_i^2$, strictly unless the pair is already capped-equal, and the sum of squares is bounded below; so re-bargaining converges, its only rest point the constrained-equal-awards allocation, again the Talmud rule and the nucleolus, with Lyapunov function $\sum_i x_i^2$. The two branches are dual: losses equalise when the estate is ample, awards when it is scarce, and the pivot $na = 1$ is where the two coincide. That the outcome is moreover a subgame-perfect equilibrium, and not merely a limit, is Dagan, Serrano, and Volij's (1997) for the bankruptcy rule. □

Remark 1 (The smooth alternative is the Nash solution)*.* The linear rival is not ad hoc. With the same primitives, the opposition value a as the disagreement point and seats as bargaining weights, the weighted Nash bargaining solution is $x_i = a + w_i(1 - na)$: a positive intercept a and a slope $1 - na$ below one, which is precisely the reduced-form small-party bias. The nucleolus and the Nash solution divide the same estate under the same claims by different logics, the one protecting the small claims up to a floor, the other splitting the surplus over disagreement smoothly. Both lie in the core, so the monitoring does not choose between them; what does is the garment. The nucleolus is the pairwise-garment-consistent division (Proposition 5); the Nash point is not. The kink identifies that garment: constrained equal awards on the half-claims. Testing the half-slope tests whether the division is the garment rather than the smooth Nash line.

Remark 2 (Neither is it the Shapley value)*.* A third division, the Shapley value of the bankruptcy game, is the random-arrival rule (O'Neill, 1982): claimants arrive in random order, each taking the least of its claim and what remains. On the three-party government above it awards 10, 5.5, 4.5, protecting the small party even more than the nucleolus does. Direction alone will not tell it from the nucleolus, for both over-reward the small. What tells them apart is the half-claim test: the nucleolus places the protected parties at exactly half their claims, and the Shapley value does not. The deeper reason the garment picks the nucleolus, and not the Shapley value or the Nash bargain, is that the nucleolus alone minimises the worst excess, the largest shortfall any

Division	Rule	The garment?	Half-claim floor?	Example
Gamson's law	proportional, $x_i = w_i$	corner a=0	no	12, 5, 3
Nash bargaining	floor + share of the surplus	no	no	8, 6.25, 5.75
Nucleolus (Talmud)	minimise the worst excess	yes	yes ($\frac{1}{2}$)	11, 5, 4
Shapley value	random-arrival rule	no	over-half	10, 5.5, 4.5

Table 1: The candidate divisions of a three-party government (twenty posts, seats 12/5/3, claims 17/10/8, floor $a = 5$). The demand game sets the over-claim a (Proposition 6), and the contested garment resolves the claims into the nucleolus; all four divisions lie in the core, so the monitoring sustains each and does not choose among them. What discriminates is the garment. Gamson's proportional rule is the garment only at the perfect-monitoring corner $a = 0$, where there is no over-claim; the Nash bargain and the Shapley value are other divisions of the estate, not the garment's. Both the nucleolus and the Shapley value over-reward the small party, but only the nucleolus seats the protected parties at exactly half their claims.

coalition can complain of. By Proposition 1 that worst excess is also the worst temptation the discipline must deter, so the garment's criterion and the discipline's hardest constraint coincide; but the monitoring sustains all three divisions, and it is the garment, not the monitoring, that selects the nucleolus among them.

7 Where the claims come from

The claim $c_i = w_i + a$ can be read from how a party bargains. A party in government forgoes its opposition value a, the office it would keep by staying out, and it brings its seats w_i to the majority. Bargaining for its due it demands both: to be made whole on the opposition it gives up, and to be paid a proportional cut of the spoils it helps secure. Its demand is $c_i = a + w_i$, a floor plus a full proportional share.

Such demands cannot all be met. They sum to $1 + na$, over-running the estate by na, the total opposition value: each party has counted its floor, but the floors compete for the one pie. That double-count is what makes the division a bankruptcy, and the size of the over-claim is the value of opposition.

There is a milder way to bargain, but it is not the one the parties play. A party might concede the double-count and demand its floor plus a share only of the surplus over all floors, $x_i = a + w_i(1 - na)$: the Nash division of Remark 1, the smooth line. That leaves each party content but leaves rents on the table, for a party can demand its full floor and still be included (Proposition 6). Pressed to that bound the demands are aggressive, $c_i = a + w_i$, and the resolution is the garment rule, not the Nash split. The choice between the two is the choice between the nucleolus and the Nash solution. The demand game makes it: the parties bring the aggressive claims, and the garment resolves them into the nucleolus.

The over-claim is set not by a sustainability ceiling but by the monitoring. Raising a lowers every reversion value $v(S) = \max\{0, 1 - \sum_{j \notin S}(w_j + a)\}$, so it makes the garment division more sustainable, not less: aspirations are not capped by what the discipline can bear. What pins a is

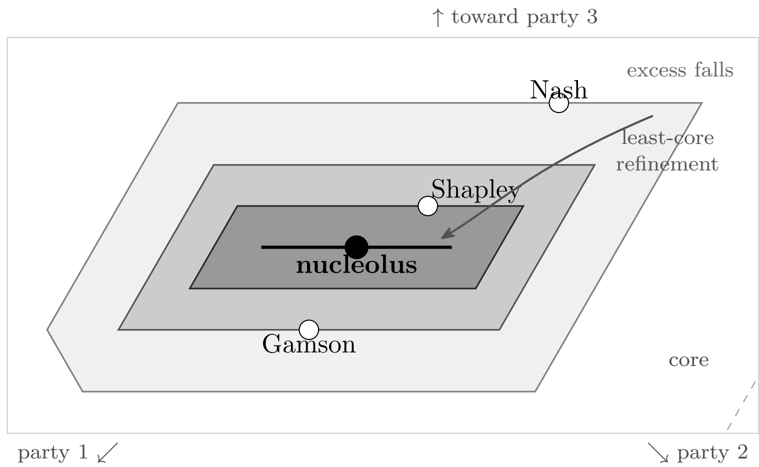

Figure 3: The nucleolus as the centre of the least-cores. The triangle is the set of divisions of the office among three parties (seats 12/5/3, claims 17/10/8); the outer dashed region is the core, and the nested regions are the least-cores, the divisions of successively smaller worst excess. This is the cooperative geometry of the nucleolus (Maschler, Peleg and Shapley, 1979): the least-cores contract to a point, the lexicographic centre, which is the contested-garment division. Gamson's proportional rule, the Nash bargain and the Shapley value all lie in the core, so the monitoring sustains them all; it is the garment, refining on the worst excess, that selects the nucleolus among them. The contraction pictured is that refinement, not the path of the monitoring, which holds the whole core. This is the coalitional counterpart of the Nash-bargaining diagram for two parties, at $n = 3$: the core in place of the bargaining frontier, and the nucleolus in place of the Nash point.

the demand game of Section 9, $a = (1 - q)L$. When monitoring is perfect, $a \to 0$, there is no over-claim and the rule is Gamson; as monitoring worsens a rises and the break widens.

8 The worth of a defection

Section 3 left the worth of a defection, $v(S)$, unspecified: the logic needed only that a defecting S secures it. The claims now fix it. When a coalition brings the government down it cannot help itself to the pie. The parties left outside hold seats a new majority may need, and each can command its claim as the price of re-forming. What S secures is the residue, once the outsiders are made whole:

$$v(S) = \max\Big\{0,\ 1 - \sum_{j \notin S} c_j\Big\}. \tag{12}$$

This is the bankruptcy game of O'Neill (1982), the estate the unit pie and the claims $c_i = w_i + a$; its nucleolus is the garment rule. The reversion values of the repeated game, grounded in re-bargaining, are a contested garment, so Proposition 2's nucleolus is Proposition 3's division. The two are one object seen from two sides: Section 4 from the discipline that sustains it, Section 5 from the claims it resolves.

Equation (12) is also the answer to what a deviation is when the object is an allocation. With two parties the object is a single line, and a deviation is one party withdrawing to what it holds alone: a single temptation on each side, and the division the discipline holds is where the two

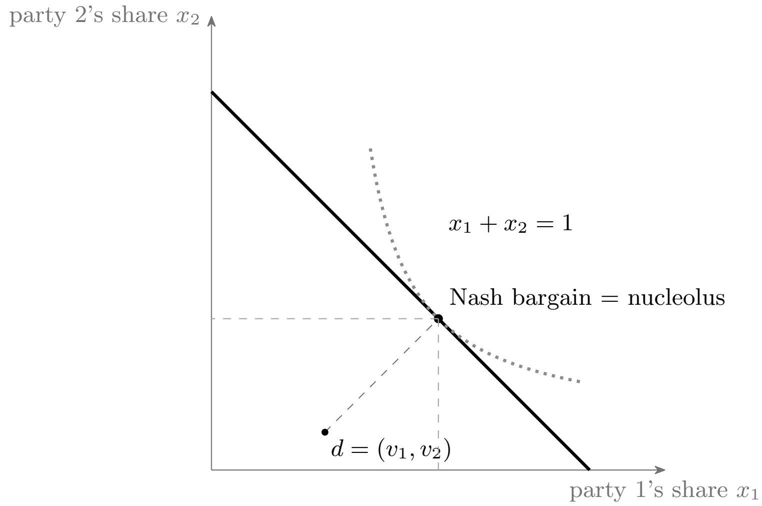

Figure 4: Corollary 1: two parties as a Nash bargain. With two parties the office is divided along the frontier $x_1 + x_2 = 1$. Taking the singleton worths (v_1, v_2) as the disagreement point d, the nucleolus is the symmetric Nash bargaining solution: it splits the surplus over disagreement equally, $x_i = \frac{1}{2} + \frac{1}{2}(v_i - v_j)$, at the point where the Nash-product contour $(x_1 - v_1)(x_2 - v_2)$ (dotted) is tangent to the frontier. Its non-cooperative foundation is Rubinstein's (1982) alternating offers, and this is the division two parties retrieve. For $n > 2$ the nucleolus and the Nash bargain part (Remark 1).

are equal. An allocation has no such single dimension. A deviation is a coalition withdrawing, and there is one for every coalition S; each concedes the parties it leaves their claims and takes the rest, as in (12). The discipline must deter them all, and the allocation that equalises the worst of these temptations, coalition by coalition, is the nucleolus. The line is the case $n = 2$: the only deviations are the two singletons, the nucleolus is the split that equalises their excesses, and that split is the division two parties retrieve.

Corollary 1 (The case of two parties)**.** *With two members the only proper coalitions are the singletons, and the nucleolus is the division that equalises their excesses, $x_i = \frac{1}{2} + \frac{1}{2}\big(v(\{i\}) - v(\{j\})\big)$. Taking the singleton worths as disagreement points, this is the symmetric Nash bargain, whose non-cooperative foundation is the alternating-offers bargaining of Rubinstein (1982): the division two parties would strike. The coalition's contested garment generalises it to $n > 2$.*

Proof. With $n = 2$ the proper coalitions are the singletons $\{1\}$ and $\{2\}$, and the nucleolus minimises the larger of the excesses $e(\{i\}, x) = v(\{i\}) - x_i$ subject to $x_1 + x_2 = 1$. Raising x_i lowers $e(\{i\}, x)$ and raises $e(\{j\}, x)$, so the minimax is attained where the two are equal, $v(\{1\}) - x_1 = v(\{2\}) - x_2$, giving $x_i = \frac{1}{2} + \frac{1}{2}\big(v(\{i\}) - v(\{j\})\big)$. With disagreement point $(v(\{1\}), v(\{2\}))$ this is the symmetric Nash bargaining solution, the equal split of the surplus over disagreement. □

That the deviation concedes each outsider its claim also disciplines the claim's shape. One number must do two jobs: what a party is conceded when it is left behind must equal what it claims when it stays. Write that number as a seat share plus an increment, $c_i = w_i + a_i$. The seat share is the competitive floor an outsider commands in re-formation, as in Morelli; the increment $a_i = c_i - w_i$ is the premium it holds for its standing. Two things then close the account. The increment is common across parties, $a_i = a$, the one simplification, and it is what yields a single estimable parameter and the clean half-slope kink. And the increment is positive only because monitoring is imperfect: the demand game of the next section shows that perfect monitoring competes it to zero and returns Morelli's proportional shares, while noise lets it survive. The additive claim is the consistency of the deviation; the size of its increment is the monitoring.

The grounding settles what troubled the small party. Its claim carries a, but a is an aspiration, not a threat it can carry out. A party that holds no majority alone secures nothing alone: by (12) its standalone worth is $v(\{i\}) = \max\{0, 1 - \sum_{j \neq i} c_j\}$, which is zero for any party the others can govern without. The small party of Section 7 claims eight, can threaten with nothing, and is awarded four: more than it could ever hold out for, less than it asked. It divides below the value it names for opposition not despite the discipline but because that value was never enforceable; a pivot is not an island. Only a party large enough that the rest fall short of the pie without it, $\sum_{j \neq i} c_j < 1$, carries a positive threat, and there the reversion binds. This is why a enters as a claim and not as a floor: it is the standing a party asks to be paid for, not an outside option it can take.

What (12) still takes on trust is that the outsiders command exactly their claims $c_j = w_j + a$ in re-bargaining. That the residue is the reversion value is the reading; that the claims are the equilibrium demands is the theorem the demand game is built to prove. That is the task of the next section.

9 The demand game

The claims themselves can be part of a game. We deploy Morelli's demand bargaining and add to it the monitoring described above (demand bargaining yields proportional payoffs in majority games when parties move in decreasing order of weight, Montero and Vidal-Puga, 2011, qualifying Morelli, 1999). Parties announce the shares they require; a government forms from a winning coalition whose demands its division can sustain; a party that demands too much is passed over for cheaper partners. In Morelli's frictionless version that threat of exclusion competes demands down to proportional shares, and the outcome is Gamson's law.

Imperfect monitoring blunts the threat. To exclude a party and form an alternative is no longer costless: the alternative government must itself be held under the same noisy signal, and where it is harder to hold the threat to walk to it is weaker. A party can then demand more than its proportional share without being dropped, up to the point where its excess would break sustainability. Demands over-run proportionality, and the size of the over-run is set by how weak the exclusion threat is, which is to say by how poor the monitoring is.

Two things follow. As monitoring becomes perfect the alternative coalitions are easy to hold, the exclusion threat is sharp, over-claiming is competed away, and the demand game returns Morelli's proportional division: Gamson in the limit. As monitoring worsens the over-run grows and the demands become a contested garment; its sustainable resolution is the nucleolus, on the claims the demand game pins. The worse the monitoring, the wider the break from Gamson.

We can now solve for the over-run. A formateur holds a winning coalition and divides the office it controls; each member i can hold out for a premium a_i above its competitive share w_i. Each period the formateur weighs two options as flows. Conceding i's premium forgoes a_i of office that period. Excluding i triggers a fresh, frictionless re-formation, a proportional government at no premium, which the same monitoring lets stand with probability q, the reliability of the signal, and which collapses with probability $1 - q$, losing the period's office rent L. Better monitoring raises q, and with it the deterrence budget κ of Section 3.

The premium sits on a proportional base. In Morelli's (1999) frictionless demand competition the threat of exclusion drives each party's demand to its seat share w_i: no party is pivotal beyond the seats it brings, so the competitive floor is proportional. The friction of imperfect monitoring sits on that base and raises the ceiling each member can hold by a common premium, which the next result pins.

Proposition 6 (The demand game pins the increment)**.** *The largest premium a member can hold without being excluded, and so the equilibrium over-demand, is*

$$a^*(q) = \min\{(1 - q)L,\ \bar{a}(q)\}, \tag{13}$$

common across parties, where $\bar{a}(q)$ is the largest premium the deterrence budget will sustain. The equilibrium claims are $c_i = w_i + a^*(q)$, resolved into the division by the garment rule of Proposition 3. The over-demand falls in the reliability of monitoring: $a^* \to 0$ as $q \to 1$, where the division is Gamson's proportional rule, Morelli's limit; a^* rises as q falls, and the division breaks toward the protective garment, until the sustainability ceiling $\bar{a}$ binds and, below it, no government can be held.

Proof. Compare the flows. Conceding i's premium costs the formateur a_i that period, on the proportional base. Excluding i triggers a re-formation that succeeds with probability q, at no premium, and fails with probability $1 - q$, costing the period's office rent L; the expected cost of excluding is $(1-q)L$. The formateur concedes exactly when $a_i \leq (1-q)L$, and since both options discount identically the present-value comparison is the same. Under the demand competition each member holds out for the most it can, so the common premium is $a^* = (1 - q)L$ wherever the claims it implies are sustainable. When $(1 - q)L$ exceeds what the budget bears, the claims violate (6), no government holds them, and the sustainable ceiling $\bar{a}(q)$ binds in its place, defined by $\varepsilon^*(a) = \kappa$; the least-core value $\varepsilon^*(a)$ increases in the over-claim a, so this crossing is unique.

Both q and κ rise with monitoring, so a^* falls; at $q = 1$ exclusion is certain, $a^* = 0$, and the claims are proportional. The garment resolution of $c_i = w_i + a^*$ is the deviation of Section 8. $\square$

Remark 3 (Why the increment is common)*.* The premium $(1 - q)L$ turns on the reliability of monitoring and the office at stake, both properties of the system and not of the party. The demand game therefore delivers a common increment, and that is what makes the claims additive, $c_i = w_i + a$, with the single parameter of Section 7. The additive form is not assumed but produced: the consistency of the deviation makes the claim a seat share plus an increment, and the demand game makes the increment common.

The demand game explains the regularity. Gamson's law is its frictionless limit, the microfoundation Morelli supplied; imperfect monitoring is the break, and the garment is what the break leaves.

10 Taking the division to data

The theory's central prediction is falsifiable, and it parts from the proportional benchmark at the shape of the division. Gamson's law is a straight proportional line; the contested garment is convex, flooring small partners and taxing large ones, a kink of lower slope one half. We take the prediction to the coalition portfolio record, the 910 governing parties of the 308 governments assembled by Martin and Vanberg (2026), and test it as Diermeier and Merlo (2004) test the rule that selects a formateur, by a non-parametric goodness-of-fit statistic. Their contest is proportional selection against a rule that favours the large; ours is the proportional rule of Gamson against the convex contested garment. We bin the governing parties by within-coalition seat share and, in each bin, compare the observed mean portfolio share to a rule's prediction, forming the Wald statistic

$$Q = (\hat{m} - p)' \, \hat{V}^{-1} \, (\hat{m} - p) \;\sim\; \chi^2_H, \tag{14}$$

distributed chi-square on the number of bins H, with $\hat{m}$ the vector of binned means, p the rule's prediction, and $\hat{V}$ their covariance clustered by government. Proportionality sets p_h to the mean seat share in bin h; the garment sets it to the contested-garment share of the coalition's claims.

The proportional rule is decisively rejected, and the garment is not. Pooling the 910 governing parties of the 308 governments in the Martin and Vanberg (2026) data into five seat-share bins, the smallest partners hold far more office than their seats warrant and the largest far less: Gamson's rule gives $Q = 286$ ($p < 10^{-50}$), while the garment gives $Q = 9.1$ ($p = 0.11$) and its prediction tracks the observed bin means almost exactly (Figure 5). Country by country the finding is the same (Table 2): proportionality is rejected in twelve of the fourteen democracies with enough governments to test, the garment in only one, and the two exceptions to the first

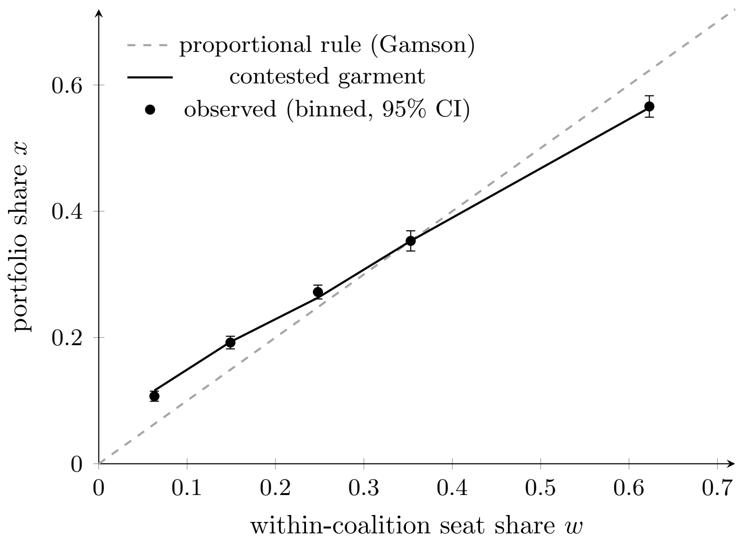

Figure 5: Coalition division against the proportional rule. Mean portfolio share against within-coalition seat share, in five bins of the Martin and Vanberg (2026) data (308 governments), with 95% confidence intervals clustered by government. The dashed line is Gamson's proportional rule, $x = w$; the solid line is the contested garment. Small partners hold more office than the proportional rule allows and large partners less; the garment tracks the observed means, while proportionality is rejected (Table 2).

are the smallest samples. Coalition division protects small partners in the convex manner of the contested garment, and departs sharply from the proportional rule.

The result survives two further checks. One asks what the division tracks, the seats a party brings or the voting power those seats confer; the other asks whether some rival claims rule fits the observed shares better than the garment does.

Seats, not power. The garment and Gamson's law both make the division track seats. A natural rival makes it track voting power instead, a party's Shapley–Shubik index in the legislative game rather than its seat share, in the spirit of a Shapley reading of the law (Demyanenko and La Mura, 2023). We race the two on the matched coalitions, computing each governing party's Shapley–Shubik power over the full parliamentary seat distribution. Regressing within-coalition portfolio share on seat share gives a slope of 0.85 ($R^2 = 0.88$); on Shapley–Shubik power alone, 0.78 ($R^2 = 0.79$); with both entered together, seats carry a coefficient of 0.82 and power an insignificant 0.03. Portfolios track the seats a party brings, not the pivotality those seats confer. This is a further standing anomaly, seats rather than power (Warwick and Druckman, 2006), and the account rationalises it directly: the discipline conditions on the salient, contractible signal, the seats a party delivers, not on a contested power index.

Which rule fits. The second half of the Diermeier and Merlo (2004) design fits each bargaining foundation to the observed division and keeps the one closest to the data. We fit, government by government, the single increment a that best matches each government's shares, and compare

		Proportional (Gamson)		Contested garment	
Country	Govts	Q	p	Q	p
Austria	20	20.1	< 0.001	0.0	0.99
Belgium	32	20.8	< 0.001	0.1	0.99
Denmark	23	79.1	< 0.001	1.8	0.62
Finland	41	12.5	0.006	1.5	0.69
France	29	11.6	0.009	4.6	0.20
Germany	25	36.5	< 0.001	0.0	0.99
Greece	6	7.6	0.056	11.8	0.008
Iceland	19	50.1	< 0.001	1.2	0.54
Ireland	12	24.8	< 0.001	4.7	0.19
Italy	38	211.6	< 0.001	6.5	0.09
Luxembourg	13	3.6	0.16	0.3	0.87
Netherlands	23	16.1	0.001	4.6	0.21
Norway	11	45.2	< 0.001	0.1	0.99
Sweden	10	38.4	< 0.001	0.0	0.99
Pooled	308	286.0	< 0.001	9.1	0.11

Table 2: A goodness-of-fit test of the division rule, after Diermeier and Merlo (2004). For each country, and pooled, the Wald statistic Q of (14) and its p-value for the proportional rule of Gamson and for the contested garment, from binned portfolio shares (three bins per country, five pooled) with covariance clustered by government. Proportionality is rejected in twelve of the fourteen countries, the garment in one. Martin and Vanberg (2026) data.

the mean absolute deviation (Table 3). Both claims rules, the contested garment of Aumann and Maschler (1985) and the Shapley random-arrival rule of O'Neill (1982), fit far closer than raw proportionality: the garment beats Gamson in all 259 of the governments where the two differ, on a sign test and a Wilcoxon signed-rank alike ($p < 0.001$). Between the two claims rules there is nothing to choose. The garment and the Shapley rule are statistically indistinguishable, on the sign test ($p = 0.26$) and the Wilcoxon ($p = 0.78$), their predictions coinciding in most governments. Two things follow. The garment and Shapley each carry the one free parameter a that proportionality lacks, so part of their common advantage over Gamson is flexibility. And overall fit is not what separates the garment from its convex neighbours: the goodness-of-fit test above rejects the proportional rule in the garment's favour, but does not, on fit alone, tell the garment from another claims rule of the $c_i = w_i + a$ family. What the structural comparison shows is narrower: coalition division is a claims problem, described by the family better than by proportionality, with the garment its best-fitting member.

Bargaining foundation	Division rule	Free incr.	MAD
Demand bargaining (Morelli, 1999)	proportional (Gamson)	0	0.069
Random-order arrival (O'Neill, 1982)	Shapley claims	1	0.023
Pairwise garment (Aumann–Maschler, 1985)	contested garment	1	**0.023**

Table 3: Structural model-selection over bargaining foundations, after Diermeier and Merlo (2004), on the 308 governments of Martin and Vanberg (2026). Each foundation implies a division rule; the two claims rules fit the single increment a per government, the proportional rule none. Entries are the mean absolute deviation of predicted from observed within-coalition portfolio shares, lower being closer. The contested garment is selected (in bold). The ordering replicates on the WhoGov data.

The theory is thus tested on data, and holds. Coalition division follows the convex contested garment; the proportional rule is rejected, pooled and country by country; and the signal the division conditions on is seats, not power, as the account requires.

11 Conclusion

The nucleolus is the division a monitored group sustains. A coalition's gain from defecting is its cooperative excess, so the divisions a repeated discipline can hold are the strong cores of the game. The binding constraint is the sharpest temptation, the largest excess, and the nucleolus is the division that makes it smallest: the most robust of the sustainable divisions. Read as a claims problem, the division is the contested garment of the Talmud, which departs from Gamson's proportional rule at a kink whose lower slope is exactly one half. Imperfect monitoring sizes the over-claim, and with it the size of that break; Gamson is the perfect-monitoring corner. Two steps are reduced-form by choice, and neither affects the result: the non-cooperative implementation of the garment (Serrano, 1995; Dagan, Serrano, and Volij, 1997), and the selection of the nucleolus within the least core by the pairwise garment.

Nothing in the argument is about parties. Any group that divides a fixed surplus and must hold it over time under imperfect monitoring meets the same discipline and retrieves the same contested garment: a cabinet dividing influence, a cartel a market, a board its seats, a federation its revenues. How a group's monitoring reliability arises from what it can observe, and what the account implies for which coalitions form and how long they last, are left to further work.

References

[Abreu and Pearce(2007)] Abreu, D., and D. Pearce (2007). Bargaining, reputation, and equilibrium selection in repeated games with contracts. *Econometrica* 75(3), 653–710.

[Abreu et al.(1990)] Abreu, D., D. Pearce, and E. Stacchetti (1990). Toward a theory of discounted repeated games with imperfect monitoring. *Econometrica* 58(5), 1041–1063.

[Acharya et al.(2025)] Acharya, A., E. Lipnowski, and J. Ramos (2025). Political accountability under moral hazard. *American Journal of Political Science*, forthcoming.

[Aumann(1959)] Aumann, R. J. (1959). Acceptable points in general cooperative n-person games. In *Contributions to the Theory of Games IV*, 287–324. Princeton: Princeton University Press.

[Aumann and Maschler(1985)] Aumann, R. J., and M. Maschler (1985). Game theoretic analysis of a bankruptcy problem from the Talmud. *Journal of Economic Theory* 36(2), 195–213.

[Austen-Smith and Banks(1988)] Austen-Smith, D., and J. S. Banks (1988). Elections, coalitions, and legislative outcomes. *American Political Science Review* 82(2), 405–422.

[Baron and Ferejohn(1989)] Baron, D. P., and J. A. Ferejohn (1989). Bargaining in legislatures. *American Political Science Review* 83(4), 1181–1206.

[Bernheim et al.(1987)] Bernheim, B. D., B. Peleg, and M. D. Whinston (1987). Coalition-proof Nash equilibria I: Concepts. *Journal of Economic Theory* 42(1), 1–12.

[Binmore et al.(1986)] Binmore, K., A. Rubinstein, and A. Wolinsky (1986). The Nash bargaining solution in economic modelling. *RAND Journal of Economics* 17(2), 176–188.

[Dagan et al.(1997)] Dagan, N., R. Serrano, and O. Volij (1997). A noncooperative view of consistent bankruptcy rules. *Games and Economic Behavior* 18(1), 55–72.

[Demyanenko and La Mura(2023)] Demyanenko, N., and P. La Mura (2023). Gamson–Shapley laws: A formal approach to parliamentary coalition formation. *Humanities and Social Sciences Communications* 10, 710.

[Dewan(2026)] Dewan, T. (2026). Splits as price wars: party discipline under imperfect monitoring. Working paper, London School of Economics.

[Diermeier and Merlo(2004)] Diermeier, D., and A. Merlo (2004). An empirical investigation of coalitional bargaining procedures. *Journal of Public Economics* 88(3–4), 783–797.

[Friedman(1971)] Friedman, J. W. (1971). A non-cooperative equilibrium for supergames. *Review of Economic Studies* 38(1), 1–12.

[Fudenberg et al.(1994)] Fudenberg, D., D. Levine, and E. Maskin (1994). The folk theorem with imperfect public information. *Econometrica* 62(5), 997–1039.

[Fudenberg and Maskin(1986)] Fudenberg, D., and E. Maskin (1986). The folk theorem in repeated games with discounting or with incomplete information. *Econometrica* 54(3), 533–554.

[Gamson(1961)] Gamson, W. A. (1961). A theory of coalition formation. *American Sociological Review* 26(3), 373–382.

[Levy(2004)] Levy, G. (2004). A model of political parties. *Journal of Economic Theory* 115(2), 250–277.

[Green and Porter(1984)] Green, E. J., and R. H. Porter (1984). Noncooperative collusion under imperfect price information. *Econometrica* 52(1), 87–100.

[Gul(1989)] Gul, F. (1989). Bargaining foundations of the Shapley value. *Econometrica* 57(1), 81–95.

[Martin and Vanberg(2026)] Martin, L. W., and G. Vanberg (2026). Refining Gamson: The isometric log-ratio transformation and portfolio proportionality in multiparty governments. *Political Analysis* (forthcoming).

[Montero(2006)] Montero, M. (2006). Noncooperative foundations of the nucleolus in majority games. *Games and Economic Behavior* 54(2), 380–397.

[Montero and Vidal-Puga(2011)] Montero, M., and J. Vidal-Puga (2011). Demand bargaining and proportional payoffs in majority games. *Games and Economic Behavior* 71(2), 395–408.

[Morelli(1999)] Morelli, M. (1999). Demand competition and policy compromise in legislative bargaining. *American Political Science Review* 93(4), 809–820.

[Maschler et al.(1979)] Maschler, M., B. Peleg, and L. S. Shapley (1979). Geometric properties of the kernel, nucleolus, and related solution concepts. *Mathematics of Operations Research* 4(4), 303–338.

[Moreno-Ternero et al.(2022)] Moreno-Ternero, J. D., M.-H. Tsay, and C.-H. Yeh (2022). Strategic justifications of the TAL family of rules for bankruptcy problems. *International Journal of Economic Theory* 18(1), 92–102.

[Nash(1953)] Nash, J. F. (1953). Two-person cooperative games. *Econometrica* 21(1), 128–140.

[O'Neill(1982)] O'Neill, B. (1982). A problem of rights arbitration from the Talmud. *Mathematical Social Sciences* 2(4), 345–371.

[Perry and Reny(1994)] Perry, M., and P. J. Reny (1994). A noncooperative view of coalition formation and the core. *Econometrica* 62(4), 795–817.

[Porter(1983)] Porter, R. H. (1983). A study of cartel stability: The Joint Executive Committee, 1880–1886. *Bell Journal of Economics* 14(2), 301–314.

[Ray and Vohra(1997)] Ray, D., and R. Vohra (1997). Equilibrium binding agreements. *Journal of Economic Theory* 73(1), 30–78.

[Ray and Vohra(1999)] Ray, D., and R. Vohra (1999). A theory of endogenous coalition structures. *Games and Economic Behavior* 26(2), 286–336.

[Ray and Vohra(2024)] Ray, D., and R. Vohra (2024). Nash bargaining with coalitional threats. Working paper, Brown University.

[Riker(1962)] Riker, W. H. (1962). *The Theory of Political Coalitions.* New Haven: Yale University Press.

[Rubinstein(1982)] Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica* 50(1), 97–109.

[Schmeidler(1969)] Schmeidler, D. (1969). The nucleolus of a characteristic function game. *SIAM Journal on Applied Mathematics* 17(6), 1163–1170.

[Serrano(1995)] Serrano, R. (1995). Strategic bargaining, surplus sharing problems and the nucleolus. *Journal of Mathematical Economics* 24(4), 319–329.

[Serrano(2005)] Serrano, R. (2005). Fifty years of the Nash program, 1953–2003. *Investigaciones Económicas* 29(2), 219–258.

[Warwick and Druckman(2006)] Warwick, P. V., and J. N. Druckman (2006). The portfolio allocation paradox: An investigation into the nature of a very strong but puzzling relationship. *European Journal of Political Research* 45(4), 635–665.