When the Buy–Sell Clause Fails

Co-ownership dissolution, from *gud o agud* to the custody of wards

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Abstract

Partnership and joint-venture agreements routinely provide for dissolution through a *buy–sell* ("Texas shootout") clause: one owner names a single price and the other chooses to buy or sell at it. We trace the device to its Talmudic ancestor, *gud o agud* (*Bava Batra* 13a), whose codified *halakha* specifies instruments the modern clause omits – a valuation floor, a solvency requirement, a waiting period, and a standing dispute over whether dissolution may be *compelled* or requires *consent*. We model the clause and show it cannot simultaneously dissolve efficiently, protect each owner's outside option, and allocate to the higher-valuer; the codified instruments are partial fixes for the three failures. When value is non-transferable the clause degenerates – to wealth dominance if the asset is alienable, to no transaction if it is not. Our central result concerns objects that have welfare of their own – a child, a holy site – which we call *wards*: here the buy–sell is displaced by guardianship under a standard, and we show that market-inalienability is, for a ward, a welfare result and not only a moral claim. This reframes King Solomon's problem – the mechanism-design literature that bears its name allocates the child as a prize to the highest valuation, whereas the judgment screens for the fit guardian by what each claimant will surrender. The argument gives a positive-economics account of why some co-owned assets are dissolved by a priced clause and others are handed to a court.

1 Introduction

When co-owners of a closely held firm, a partnership, or a joint venture fall out, the contract that binds them often contains a *buy–sell* clause – in practice a "Texas shootout" or "shotgun" provision – to break the deadlock. Its logic is disarmingly simple: one owner names a single

price, and the other elects either to buy the namer out at that price or to sell to the namer at it. Because the namer does not control which side of his own offer he will end up on, he is disciplined toward a price he would accept as buyer or as seller – a *self-correcting valuation* that curbs both lowballing and inflation (Brooks et al., 2010; de Frutos and Kittsteiner, 2008). The clause is a stock device of incomplete-contract practice: an ex-ante rule for an ex-post contingency – the breakdown of co-operation over an asset more valuable whole than divided – that the parties cannot easily contract on in detail (Grossman and Hart, 1986; Hart and Moore, 1990); the clause is the governance structure they set up for that contingency (Williamson, 1979).

The device is far older than its Texan name. The Babylonian Talmud states it at *Bava Batra* 13a as *gud o agud* – "name a price [and buy from me], or I will name a price [and buy from you]" – as the remedy for jointly held property that cannot be fairly divided. The sugya and its codification carry a richer institutional specification than the modern clause: a rule that the named price may not fall below the asset's value, a requirement that the initiator be able to buy the other out himself, a waiting period for the respondent, and – the live controversy – a dispute over whether a partner may *compel* the procedure (Rav Yehuda) or whether, absent agreement, "the partnership continues" (Rav Naḥman). Read as contract design, the law had already engineered the instruments the economics literature would later study in isolation.

This paper does three things. First (§§2–5), it models the clause and shows that no version of it can simultaneously (D) dissolve whenever efficient, (V) leave no owner worse off than continued co-ownership, and (A) allocate the asset to the owner who values it most. The codified instruments map onto this impossibility as partial fixes: the *floor* answers the allocational failure, the *solvency* requirement restores the role-symmetry on which honest pricing depends, and the *compel-versus-consent* dispute is precisely the choice of which of (D) and (V) to sacrifice. The reading's first yield is that codified halakha already carries the design instruments – floor, solvency, compel-versus-consent – that the modern buy–sell omits. Second (§7), it shows that when value is non-transferable – sacred, or identity-bound – the clause degenerates: to allocation by wealth if the asset is alienable, to no transaction at all if it is not. Third, and centrally (§8), it identifies the class of objects for which the priced clause is not merely imperfect but wrong: objects with welfare of their own – a child, a holy site – which we call *wards*. For a ward the buy–sell is displaced by guardianship under a welfare standard, and barring the priced channel (market-inalienability) is welfare-improving for the ward, not only a moral stance. The result supplies a positive-economics rationale for why family courts allocate custody, and why holy places are held in trust, rather than auctioned between the claimants. It also reframes a classic problem: the mechanism-design

literature that carries King Solomon's name (Glazer and Ma, 1989; Moore, 1992; Perry and Reny, 1999) treats the disputed child as a prize won by the highest valuation, whereas the judgment screens for the fit guardian by what each claimant will surrender. Correctly specified, Solomon's is a ward problem, not a prize problem.

This paper is one of a series that reads the Talmud as a source of formal political economy. The precedent is Aumann and Maschler (1985), who show that a Mishnaic division of an estate encodes the nucleolus of the associated coalitional game. We take the program a step further. Where that work recovered a solution to a problem the analyst poses, we read the sugya as the record of an institution: a procedure, legislated in precise terms and defended in argument, that responds to a strategic problem of information, commitment, or division. The task is to recover the environment the procedure presupposes, and to ask what, in that environment, the procedure achieves. The corpus rewards the exercise. It is a vast and argued record of institutional design under adversarial conditions; its mechanisms are stated finely enough to be modelled; and its disputes between named authorities often isolate the very trade-off a modern analysis would name. The claim is not that the sages anticipated our theorems. It is that the tradition has legislated, and reasoned about, institutions whose logic formal theory can make explicit, evaluate, and carry to settings far from their origin.

2 The institution and its lineage

The modern clause. Buy–sell provisions appear throughout partnership, LLC, and close-corporation agreements as deadlock- and exit-mechanisms. The economics is by now well understood. Cramton et al. (1987) show that an equal (50/50) partnership can in principle be dissolved efficiently by a well-chosen mechanism; McAfee (1992) gives simple mechanisms that do well; de Frutos and Kittsteiner (2008) show the named-price buy–sell is efficient when the proposer is the partner who perceives herself equally likely to buy or to sell; and Brooks et al. (2010) show that, with common-value uncertainty, owners are "gun shy" – they avoid naming buy–sell prices because doing so cedes informational surplus – so that assigning *trigger rights* (the power to force the other to name a price) raises efficiency. Our model recovers these forces; the contribution here is not the mechanism but its reading as a legal institution with a deeper design, and the identification of where it ceases to be the right institution at all.

The ancestor and its trigger. The device is stated in the sugya at *Bava Batra* 13a – *gud o agud*, "name a price [and buy from me], or I name a price [and buy from you]" – as the remedy for jointly held property that cannot be fairly divided. What routes a dispute

to it is the divisibility test of the same sugya (*Bava Batra* 13b): an item may be physically divided only if each part still bears *the name* of the whole; if it does not – half a courtyard, half a vessel, half a slave is not the thing – "its monetary value is assessed" and *gud o agud* applies. This *shem* (name) test is our indivisibility primitive: the clause is for assets that cannot be split without ceasing to be what they are – the property-rights question of when a jointly held thing is one asset rather than separable claims (Hansmann and Kraakman, 2000). The sugya already feels the squeeze of §6: Rabban Shimon ben Gamliel objects that a partner may have "no money to give… and is not at ease with a gift" (*Bava Batra* 13b) – the cash-poor party for whom a priced exit is no exit at all.

The codified design. The practical *halakha* (in the rishonim and the codes, principally Shulchan Arukh, *Choshen Mishpat* 171, the laws of dividing partners) specifies more than the bare clause: the named price may not be below the asset's true value; the initiator must have the means to buy the other out himself; and the respondent has a fixed window (thirty days) to decide – the last three broadly attributable to the glosses of R. Isserles (the Rema).[1] Above these sits the amoraic dispute (*Bava Batra* 13a): Rav Yehuda (ruled as *halakha* by Ameimar) holds that a partner may compel *gud o agud*; Rav Naḥman holds there is no such compulsion and "the partnership continues," a position Rava follows in a hard case. The codes read the two not as flat contradiction but contextually: compulsion holds where the parties are symmetric – each able to occupy either role, namer or chooser – while Rav Naḥman's "the partnership continues" governs where that symmetry fails (the half-slave, who can say "I will set a price" but not "you set a price"), the very role-symmetry our solvency requirement restores. Compulsion and consent are thus the halakha's own names for our regimes Y and N. As we show, each codified instrument answers a specific failure of the bare clause – the law is, in effect, a contract-design menu, assembled case by case, for the impossibility of §5.

This is distinct from the Talmud's other, more famous division problem – the bankruptcy or claims problem of the contested garment (*Bava Metzia* 2a) and the estate among creditors (*Ketubot* 93a), first formalised by O'Neill (1982) and whose divisions Aumann and Maschler (1985) identify with the *nucleolus*. That is division of a contested sum that falls short; ours is partition of a jointly owned indivisible whole. The two should not be conflated.

[1]These instruments are distributed across *Choshen Mishpat* 171; we do not pin a *se'if*, as the sub-section numbering varies across editions.

3 The game

Environment. Two owners $i \in \{1, 2\}$ jointly hold one *shem*-indivisible asset, each owning a half-share. Owner i's value for sole ownership is v_i, privately known and drawn independently from F_i with density f_i on $[\underline{v}, \overline{v}]$. Owner i has cash $w_i \geq 0$. If they do not dissolve, continued joint use yields each owner an outside payoff q_i (the gemara's "work for this one one day, the other two days"); sole ownership is more efficient than shared, so there are gains from dissolving. Measuring both as stocks, continuation yields the pair $q_1 + q_2$ while sole ownership by i yields surplus v_i (the winner compensating the loser by transfer); dissolution is then *efficient* – and should run to $\arg\max_i v_i$ – exactly when $\max_i v_i > q_1 + q_2$, which we assume can occur.

The clause (*gud o agud*). A rule designates one owner the *namer*, the other the *chooser*.

> **Stage 1.** The namer announces a price $p \geq 0$ for the whole asset ($p/2$ per half-share).
>
> **Stage 2.** The chooser either *buys* the namer's half at $p/2$ (and owns the whole) or *sells* her half at $p/2$ (and the namer owns the whole).

Final payoffs, each owner beginning from a half-share:

$$\text{2 buys:} \quad \Big(U_1, U_2\Big) = \Big(\tfrac{p}{2},\ v_2 - \tfrac{p}{2}\Big); \qquad \text{2 sells:} \quad \Big(U_1, U_2\Big) = \Big(v_1 - \tfrac{p}{2},\ \tfrac{p}{2}\Big).$$

Two regimes – the amoraic dispute.

- **Regime Y (compulsion, Rav Yehuda).** Either owner may unilaterally trigger the clause; the other is bound. Forced exchange. (This is the assignment of a *trigger right* in the sense of Brooks et al., 2010.)
- **Regime N (consent, Rav Naḥman).** The clause runs only by mutual consent; otherwise "the partnership continues." No forced exchange.

4 Equilibrium

The game has incomplete information and an observable first-stage action, so the concept is **Perfect Bayesian Equilibrium**: a namer strategy $p^*(\cdot)$, a chooser strategy $a^*(\cdot)$, and a belief system $\mu(\cdot \mid p)$ over the namer's type, sequentially rational and Bayes-consistent on the equilibrium path.

Remark 1 (Belief-freeness)**.** The chooser's payoff does not depend on the namer's type v_1. Hence her best response is *belief-independent*,

$$a^*(p, v_2) = \text{buy} \iff v_2 \geq p \;\text{ and }\; w_2 \geq \tfrac{p}{2},$$

so μ does no work: off-path prices cannot be "refined" and no refinement is needed. The PBE reduces to backward induction – a dominant action at Stage 2, a Bayesian-optimal price at Stage 1. The two are pinned down from distinct sources: the chooser's dominant action leaves no role for off-path beliefs, and the namer's optimal price is unique under the standard regularity condition that $p - \frac{1-F_2(p)}{f_2(p)}$ is increasing, so the equilibrium outcome is essentially unique. This belief-robustness is a genuine virtue of the clause: the outcome does not hang on what the ceding owner infers about the other's valuation, which is one reason such a blunt device is used in place of a finer mechanism.

The namer, facing $\Pr(\text{buy}) = 1 - F_2(p)$ (with a solvent chooser), maximises

$$U_1(p; v_1) = \tfrac{p}{2}\,[1 - F_2(p)] + \Big(v_1 - \tfrac{p}{2}\Big)F_2(p) = \tfrac{p}{2} + (v_1 - p)\,F_2(p), \tag{1}$$

with first-order condition $\frac{1}{2} - F_2(p) + (v_1 - p)f_2(p) = 0$. Truth-telling $p = v_1$ is optimal only when $F_2(v_1) = \frac{1}{2}$ – the proposer who perceives herself equally likely to buy or sell, exactly de Frutos and Kittsteiner (2008); otherwise the namer shades toward the chooser's median. The device is not strategy-proof.

5 The clause cannot do everything: a design trilemma

We ask what the clause can guarantee, over the class of mechanisms it belongs to.

Definition 1 (The *gud o agud* family)**.** A *named-price* mechanism designates one owner the namer and the other the chooser (the role assignment may be random or wealth-contingent), has the namer announce a single price $p \geq 0$, and lets the chooser buy or sell her half at $p/2$; it may carry a reserve floor and a solvency screen, and runs either by compulsion (regime Y) or by consent (regime N). The *gud o agud family* is the set of such mechanisms. It excludes mechanisms with richer message spaces or transfers contingent on both reports.

Definition 2 (Three desiderata)**.** For the dissolution clause: **(D) Efficient separation** – it dissolves whenever sole ownership yields more joint surplus than continuation, and efficient dissolutions are not blocked; **(V) Voluntariness** – every owner's interim payoff is at least q_i; **(A) Value-allocation** – conditional on dissolving, the asset goes to $\arg\max_i v_i$.

Lemma 1 (Value-allocation generically fails)**.** *For any regular F_2, no member of the* gud o agud *family attains* (A)*: the namer's price equals his value, $p^*(v_1) = v_1$, only on the null set $\{v_1 : F_2(v_1) = \frac{1}{2}\}$, so on a set of namer types of full measure the chooser's buy threshold $p^*(v_1)$ differs from the efficient threshold v_1 and the asset goes to the lower valuer with positive probability.* (A) *is a standing failure of the clause, in every regime and with deep pockets.*

Proof. From (1) the first-order condition $\frac{1}{2} - F_2(p) + (v_1 - p)f_2(p) = 0$ reads, at $p = v_1$, $F_2(v_1) = \frac{1}{2}$; so $p^*(v_1) = v_1$ only where $F_2(v_1) = \frac{1}{2}$, a single point since F_2 is strictly increasing, and $p^*(v_1) \neq v_1$ elsewhere (the regularity condition of Remark 1 makes $p^*(\cdot)$ the unique maximiser, so it is well defined). The chooser buys iff $v_2 \geq p^*(v_1)$ (Remark 1) whereas efficiency assigns the asset to owner 2 iff $v_2 \geq v_1$. The two thresholds differ off the null set, and misallocation falls on the band of v_2 between v_1 and $p^*(v_1)$: where $p^*(v_1) < v_1$ the chooser buys though she values it less, and where $p^*(v_1) > v_1$ she sells though she values it more. Either way the lower valuer wins, on an event of positive probability, independently of wealth and of the consent/compel regime. □

Proposition 1 (The clause cannot attain (D), (V) and (A))**.** *No member of the* gud o agud *family satisfies* (D)*,* (V) *and* (A) *together. Allocation* (A) *fails generically and unconditionally (Lemma 1); conditional on that standing failure, and on priors where* (D) *and* (V) *conflict, the family faces a binary default between them, fixed by the regime – consent* (N) *keeps* (V) *but loses* (D) *(low types veto efficient dissolutions: hold-up), compulsion* (Y) *keeps* (D) *but loses* (V) *(a dragged-in owner is pushed below q_i: expropriation) – and binding wealth makes the allocational failure regressive, the asset going to the wealthier rather than the higher-valuing owner (the squeeze).*

Proof. (A) fails on a full-measure set by Lemma 1. For the D–V default it suffices to exhibit a prior on which the conflict is live: the uniform example of §6 gives, on the band $\frac{1}{16} < q \leq \frac{1}{4}$, namer types $v_1 < 2\sqrt{q} - \frac{1}{2}$ who prefer continuation though dissolution is efficient; under (N) they veto efficient dissolutions ((D) fails), under (Y) the chooser triggers and drags them below q_i ((V) fails). Under binding wealth a cash-poor high-valuer cannot fund the buy-out, so the asset goes to the wealthier party, sharpening the (A)-failure into a regressive one. □

Remark 2 (The impossibility is a property of the clause, not the problem)**.** The D–V conflict is the familiar tension between ex-post efficiency and interim participation under two-sided private information (Myerson and Satterthwaite, 1983). It is not an impossibility of the environment: at equal half-shares a richer mechanism attains efficient, individually-rational dissolution (Cramton et al., 1987) – the equal-ownership case that escapes the Myerson–Satterthwaite frontier. The trilemma is therefore a property of the *named-price clause*, which

forgoes that frontier for the belief-robust simplicity of Remark 1. Read against this, the codified *halakha* is a menu of patches, each targeting one failure: the *below-value floor* caps the squeeze that worsens (A); the *solvency requirement* restores the role symmetry on which self-correcting pricing rests; and the *compel-versus-consent* dispute is the choice between sacrificing (V) (Rav Yehuda) and sacrificing (D) (Rav Naḥman). The law does not escape the tension; within the clause, it allocates the loss deliberately.

6 A worked example: uniform values, with a poor type

Let $v_1, v_2 \sim U[0,1]$ independently; owner 1 names, owner 2 chooses; the outside option is symmetric ($q_1 = q_2 = q$), and each owner's payoff is compared to his own q.

Optimal price and shading. With $F_2(p) = p$, (1) gives $\frac{1}{2} - p + (v_1 - p) = 0$, hence

$$p^*(v_1) = \frac{v_1}{2} + \frac{1}{4}. \tag{2}$$

The namer tells the truth ($p^* = v_1$) only at $v_1 = \frac{1}{2}$; high types name below value, low types above – shading toward the opponent's median $\frac{1}{2}$.

(A) fails from shading alone. The chooser buys iff $v_2 \geq p^* = \frac{v_1}{2} + \frac{1}{4}$, whereas efficiency wants the asset to owner 2 iff $v_2 \geq v_1$. Since $\frac{v_1}{2} + \frac{1}{4} \neq v_1$ off $v_1 = \frac{1}{2}$, on a positive-measure set the asset goes to the lower valuer: value-allocation is the robust casualty, failing even with deep-pocketed owners.

Payoffs. Substituting (2) into (1) yields the clean identity

$$U_1(v_1) = \Big(p^*(v_1)\Big)^2 = \Big(\tfrac{v_1}{2} + \tfrac{1}{4}\Big)^2 \in \Big[\tfrac{1}{16}, \tfrac{9}{16}\Big]. \tag{3}$$

The chooser's interim payoff, with $p^* \sim U[\frac{1}{4}, \frac{3}{4}]$, is

$$U_2(v_2) = \begin{cases} \frac{1}{4}, & v_2 \leq \frac{1}{4}, \\ v_2^2 - \frac{v_2}{2} + \frac{5}{16}, & \frac{1}{4} \leq v_2 \leq \frac{3}{4}, \\ v_2 - \frac{1}{4}, & v_2 \geq \frac{3}{4}, \end{cases}$$

bounded below by $\frac{1}{4}$: even a zero-value chooser sells for $\mathbb{E}[p^*/2] = \frac{1}{4}$.

(D) versus (V): hold-up and expropriation. The namer clears his outside option iff $U_1(v_1) \geq q$, i.e. $v_1 \geq 2\sqrt{q} - \frac{1}{2}$. Three regions in q:

- $q \leq \frac{1}{16}$: all types clear IR – (D) and (V) both hold (and (A) still fails).
- $\frac{1}{16} < q \leq \frac{1}{4}$: the chooser always wants in ($U_2 \geq \frac{1}{4} \geq q$), but namers with $v_1 < 2\sqrt{q} - \frac{1}{2}$ prefer continuation. Under **N** they veto – efficient dissolutions are blocked (*hold-up*); at $q = \frac{1}{4}$ half the namer types block. Under **Y** the chooser triggers and the low-v_1 namer is dragged below q (*expropriation*).
- $q > \frac{1}{4}$: continuation dominates on regions for both sides.

Rav Yehuda and Rav Naḥman therefore differ only on the bounded band $\frac{1}{16} < q \leq \frac{1}{4}$; outside it the regimes coincide (Figure 1).

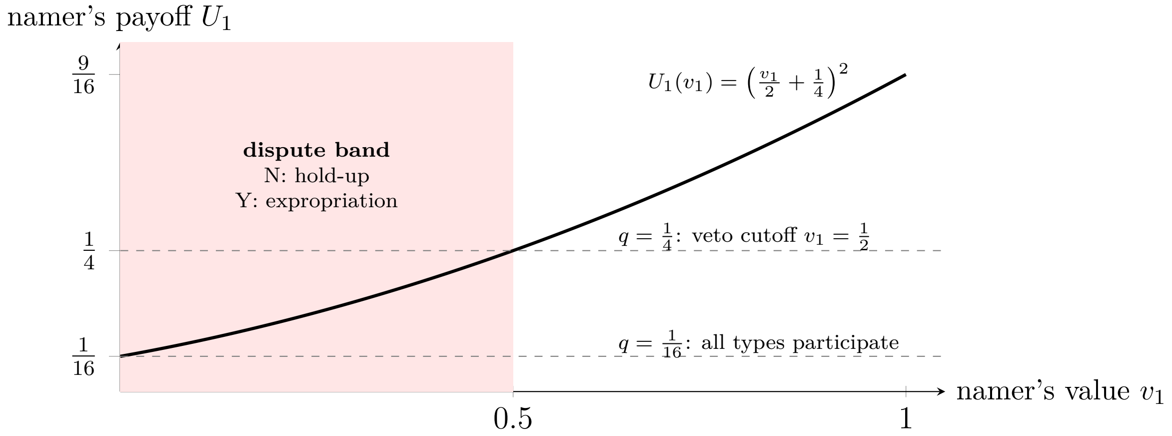

Figure 1: The namer's payoff $U_1(v_1)$ and the outside option q. For $q \leq \frac{1}{16}$ every type participates; for $q = \frac{1}{4}$ the veto cutoff is $v_1 = \frac{1}{2}$. On the shaded band the low-value namer prefers continued co-ownership, so consent (N) yields hold-up and compulsion (Y) yields expropriation. Outside $\frac{1}{16} < q \leq \frac{1}{4}$ the two regimes coincide.

The squeeze, and the floor. Now let the chooser be poor ($w_2 = 0$) with probability ρ: she cannot afford $p/2$, so she sells regardless of v_2. A high-value poor chooser is forced to cede the asset to the lower-valuing but solvent namer – value-allocation fails regressively. Worse, against an observably poor chooser the namer's optimal price is $p \to 0$: he buys her forced-sold half for almost nothing, and the expropriation is unbounded. This is the content of Rabban Shimon ben Gamliel's objection (*Bava Batra* 13b), "I have no money to give you... and I am not at ease with a gift," and it is exactly what the codified *below-value floor* caps – the missing instrument that bounds $p \to 0$ expropriation of a forced seller. Here the law's design and the model's pathology meet precisely.

7 When the price channel closes

The squeeze closes the price channel through *poverty*. The hardest assets close it differently: they are valued non-monetarily – sacred, or identity-bound – so no price clears even for the wealthy. Indivisibility by itself rarely defeats a bargain, for a side-payment or a lottery can usually divide the prize (Fearon, 1995); the hard case is the good whose sanctity resists both partition and substitution – a contested holy place (Hassner, 2003) – where *non-transferability* bites, stripping away the very side-payment that would otherwise do the dividing. The clause then degenerates.

Payoffs with an ownership premium. Let owner i holding cash m_i receive $\theta_i + m_i$ if she owns the asset and m_i if she does not, where $\theta_i > 0$ is the ownership value and w_i caps payments. *Non-transferable* value is $\theta_i \to \infty$ (θ_i exceeds every feasible transfer): ownership is lexicographically preferred to money. The asset is *alienable* if an owner will accept cash to part with it ("sell" available) and *inalienable* if parting is categorically refused ("sell" yields $-\infty$; cf. Radin, 1987).

Proposition 2 (Degeneration under non-transferable value)**.** *Take the clause in the limit $\theta_i \to \infty$.*

(i) (Alienable: wealth dominance.) *The asset ends with the wealthier owner in either role, at a price approaching $2\min\{w_1, w_2\}$ from above; the poorer is stripped of an asset no transfer compensates. (A) collapses onto wealth.*

(ii) (Inalienable: no completion.) *If both owners refuse to sell, no buy/sell profile is feasible: the clause has no outcome. Only continued joint ownership (Rav Naḥman) or unilateral seizure survive.*

Proof. With $\theta_2 \to \infty$ the value test $v_2 \geq p$ never binds, so the chooser buys iff she can afford it, $p \leq 2w_2$, else is forced to sell. The namer compares owning, worth $\theta_1 + w_1 - \frac{p}{2} \to \infty$, to cash $w_1 + \frac{p}{2}$; he prefers to own and secures it by naming any p just above $2w_2$ to force a sale (the chooser then cannot afford to buy), feasible iff $w_1 > w_2$. A poorer namer, by contrast, cannot force the richer chooser to sell, so the richer owns in either role; the forcing price approaches $2\min\{w_1, w_2\}$ from above, giving (i). For (ii), "buy" requires the other to "sell"; if neither will, no profile completes. □

Remark 3 (Three ways the price channel closes)**.** Selection fails for three distinct reasons: *poverty* (§6); *inalienability* (sacred or identity value, Proposition 2); and *interdependence* (a brand or title worth more precisely because it denies the rival, breaking private values,

so that efficient dissolution can fail even under a richer mechanism, Fieseler et al., 2003). Which closure obtains, and whether an authority can enforce continuation, determines the institution that replaces the clause (§9).

8 The ward branch: when the object has welfare

Every buy–sell treats the asset as inert – valued only through the owners. The cases for which the clause is felt to be intolerable, not merely imperfect, break that assumption. A child in a custody dispute, and a holy site, share a feature the clause cannot see: the object has its own protected interest – the child's welfare, the site's sanctity. Call such an object a *ward*. We read one Talmudic case in this light: the half-slave of §2, for whom the court refuses symmetric *gud o agud* and acts on his own account. The ward reading is our reconstruction, not the sugya's gloss; but the half-slave is at once where role-symmetry fails and where the court takes over, and that coincidence is the paper's hinge. Then the maximand is its welfare, not the claimants' valuations, and the institution shifts from transfer of ownership to guardianship under a standard.

Setup. The court's maximand is the ward's welfare, and it does not observe fitness – though it can investigate, at a cost and imperfectly. Sole custody by claimant i yields ward welfare $W_i \geq 0$, with first-best $W^* = \max_i W_i$; a home study or its like yields a noisy signal $\hat{W}_i = W_i + \varepsilon_i$ at cost $c > 0$, the errors ε_i independent and mean-zero. Each claimant has a private *desire* $v_i > 0$ for custody, private wealth w_i, and a private *welfare-concern* $\alpha_i \geq 0$ drawn independently and identically across claimants, with fitness increasing in concern, $W_i = W(\alpha_i)$, $W' > 0$: the better guardian is the one who cares more for the ward. The crux is that desire and fitness diverge: a claimant may want custody most (v_i high) yet guard it worst (α_i low), the eager but unfit parent. A buy–sell selects on desire-and-wealth and cannot see fitness; the question is what the court should do instead.

Definition 3 (Property versus ward)**.** The object is *property* if the social objective is the claimants' value (the asset is inert), and a *ward* if the objective is its own welfare W.

Proposition 3 (The ward branch)**.** *The court does not observe fitness. Let $k = k(v, w)$ be the buy–sell winner.*

(i) (The price misallocates.) *The buy–sell awards custody to the claimant of higher desire-and-wealth; since (v, w) is independent of fitness and the concerns α_i are identically distributed, the winner is the fitter guardian only by coincidence – with probability $\frac{1}{2}$ – so W^* is attained by chance.*

(ii) (Investigation beats the price.) *A court that investigates, paying c for the signals $\hat{W}_i$, allocates to* $\arg\max_i \hat{W}_i$ *and selects the fitter guardian with probability $\pi > \frac{1}{2}$, rising to one as the signal sharpens – strictly better than the price's coin flip.*

(iii) (Inalienability is ward-improving when the stake exceeds the cost.) *Barring the priced channel – direct and bundled (Mnookin and Kornhauser, 1979) – and committing to investigation raises expected ward welfare by $(\pi - \frac{1}{2})\,\mathbb{E}[\,W^* - W_{\min}\,] - c$, positive exactly when the accuracy-weighted stake of misallocation exceeds the cost of learning fitness. Market-inalienability (Radin, 1987) is then a welfare result, not only a moral claim.*

Proof. (i) The winner $k(v, w)$ is measurable in (v, w), which is independent of fitness; with α_1, α_2 identically distributed the claimants are exchangeable in W, so $\Pr\Big(k = \arg\max_i W_i\Big) = \frac{1}{2}$. (ii) Allocating to $\arg\max_i \hat{W}_i$ matches $\arg\max_i W_i$ with probability $\pi = \Pr(\arg\max_i \hat{W}_i = \arg\max_i W_i)$, which exceeds $\frac{1}{2}$ for any informative signal and rises to one as the noise vanishes. (iii) Investigation yields expected welfare $\pi\,\mathbb{E}[W^*] + (1 - \pi)\,\mathbb{E}[W_{\min}]$ and the price yields $\frac{1}{2}\mathbb{E}[W^*] + \frac{1}{2}\mathbb{E}[W_{\min}]$, so investigating, net of c, gains $(\pi - \frac{1}{2})\,\mathbb{E}[W^* - W_{\min}] - c$, positive iff the accuracy-weighted stake exceeds the cost. □

Remark 4 (Screen instead of investigate)**.** The court need not always pay c. A relinquishment test – Solomon's, §8.1 – that confronts each claimant with conceding custody to the rival or putting the ward at risk screens on welfare-concern directly: when the net gain to conceding rises in α_i (single-crossing), only the higher-α claimant concedes, and the court reaches $\arg\max_i W_i$ without verification. The choice between investigating and screening is the ward-branch counterpart of the property-versus-clause choice: pay to learn fitness, or design a test that makes the claimants reveal it.

8.1 King Solomon's problem, reframed

The screening test of Remark 4 is Solomon's. The implementation literature that bears his name (Glazer and Ma, 1989; Moore, 1992; Perry and Reny, 1999) reads the problem as allocating a *prize*, without transfers, to the claimant of highest valuation, screened by off-equilibrium threats. In that reading valuation already proxies entitlement: the true mother values the child most because she is the mother, so selecting on desire selects the rightful claimant. Our ward setup denies exactly this. Desire v_i and welfare-concern α_i are independent primitives, so the claimant who wants custody most need not be the one who guards it best; under independence the two diverge generically. A prize mechanism, screening on v, then misallocates. The judgment screens on the other variable. It awards the child to

the woman who yields rather than see it cut – the claimant who subordinates possession to the ward's life, the high-α guardian, not the high-v owner.

The screen works under one condition. Let the test confront each claimant with conceding to the rival or pressing a claim that risks the ward; pressing wins possession, worth v_i, at a welfare cost rising in α_i. When the net gain to conceding is increasing in α_i – single-crossing – only the higher-α claimant concedes, and awarding custody to the conceder reaches $\arg\max_i W_i$. This also locates the literature's known fragility. The naive test fails when the unfit claimant concedes in order to be handed custody: single-crossing breaks and mimicry returns. Repairing it is the work of the robust mechanisms (Moore, 1992; Perry and Reny, 1999; Bag and Sabourian, 2005), which restore separation through fines and off-path play; we take that machinery as given. What the property/ward distinction contributes is prior to the mechanism – the identity of the screened variable. The price sorts on desire v; the guardianship test sorts on concern α; and a ward problem is exactly one in which the two come apart.

This places the paper next to Ayres and Talley (1995), its nearest prior. They too take Solomon as their text, asking how dividing a contested entitlement – a liability rule letting either party buy the other out – facilitates Coasean trade under private information, a benefit Kaplow and Shavell (1995) dispute. But their disputed thing is property: the division serves the claimants, and success is trade reaching the higher-valuing owner. Ours is a ward. Its welfare is the maximand, not the claimants' surplus, and the screen sorts on alignment α, not valuation v. Where Solomonic bargaining divides an entitlement to move it to whoever values it most, the judgment withholds it from the market to protect a third interest – the property/ward line this paper draws.

9 The domain of the clause

The clause is the right institution in one region and the wrong one outside it. The boundary has two coordinates – whether value is transferable, and whether an authority can compel an outcome – and the ward branch cuts across them. The sorting maps onto the trichotomy of Calabresi and Melamed (1972): the buy–sell is a liability rule, while a ward is what the law renders inalienable. Whether such a liability rule facilitates efficient bargaining over a shared entitlement is the question of Ayres and Talley (1995) and Kaplow and Shavell (1995); the trilemma is our answer for the named-price family, and the D–V default is where that facilitation is paid for.

	Authority present	**Authority absent**
Value transferable	buy–sell clause selects an owner *(partnership / JV deadlock – the baseline)*	bilateral buy–sell with hold-up
Value non-transferable	frozen status quo / court allocation *(holy sites)*	contest over the boundary *(rival turf)*

Proposition 4 (The institution sorts on the primitives)**.** *The arrangement that arises is fixed by the environment: whether value is transferable, whether (failing that) the asset is alienable, whether an authority can enforce an outcome, and whether the object is property or a ward.*

(a) Property, transferable value. *The buy–sell runs (Prop. 1); an authority makes it bind and the regime fixes the D–V default, its absence leaves the same clause exposed to hold-up.*

(b) Property, non-transferable value. *If the asset is alienable the clause degenerates to wealth dominance; if inalienable no sale completes and only continuation survives (Prop. 2) – a frozen condominium under an authority, perpetual contest without one.*

(c) Ward. *The priced channel is barred and a guardian allocates under a standard – by investigation when the accuracy-weighted stake exceeds its cost, by the relinquishment screen otherwise (Prop. 3, Remark 4).*

The map is positive, not merely prescriptive: each cell names the institution the primitives produce, and the arrangements one observes – partnership buy–sells, family-court custody, trusteed holy places – sit in the cells the model assigns them.

Business deadlock – the baseline. Partnership and JV breakups are the home cell: value is transferable, an authority enforces, and the clause runs as designed – the trilemma of §5 and the trigger-rights logic of Brooks et al. (2010) are the whole story.

Child custody – the leading ward. Custody is the case that makes the bifurcation unavoidable. A child cannot be sold; the buy–sell is barred by law – the paradigm of market inalienability (Radin, 1987). The two real regimes are precisely the survivors of Proposition 2(ii): *shared custody* (continuation) and *allocation by a court* under a best-interests standard. Yet custody is bargained against the divisible marital estate, so a *shadow*

buy–sell reopens (Proposition 3(iii); Mnookin and Kornhauser, 1979) and the wealth-dominant squeeze returns – now against the child's interest. That is the positive-economics case for barring the priced channel and policing settlements: by Proposition 3(iii), inalienability protects the ward from allocation by wealth.

Holy sites – guardianship of sanctity. A contested holy place illustrates the inalienable, authority-present cell: by Proposition 2(ii) no buy–sell completes, and what survives is guardianship – a custodian, or a frozen condominium "status quo." On the ward branch the protected interest is the site's own sanctity, so the institution guards it rather than allocating between claimants. We treat the case strictly as one of formal governance.

Party names and turf – the spectrum. A party split contests the name and incumbency, worth more for denying the rival; private values fail by interdependence (Remark 3), and an electoral commission or court assigns the symbol – third-party allocation again. Rival turf is the authority-absent corner, where "the partnership continues" resolves into perpetual contest rather than peaceful condominium.

The shared arena – condominium by space and time. Not every indivisible turf collapses into contest. Milan's two clubs, A.C. Milan and Internazionale, share one ground, the San Siro, that neither will quit and neither can buy from the other – a *shem*-indivisible asset, identity-bound on both sides, on which the priced clause degenerates (Proposition 2). They run no buy–sell. They sustain instead the standing arrangement that Rav Naḥman's "the partnership continues" names, dividing the indivisible along two axes at once. In space, each keeps its own end – Milan the Curva Sud, Inter the Curva Nord – so the ground bears both identities without being cut. In time, the fixture list alternates which club is at home, the visitors taking the other's end for the day. A whole that cannot be split, and a value neither will sell, is shared not by a price but by a rule that allocates place and turn. The arrangement is sustained under a governing authority, the league – the authority-present corner where the model expects condominium rather than contest – and it is the paper's illustration of continuation done well, a peace the priced clause cannot itself reach.

The Talmud's own bifurcation. On our reading the sugya draws the line this paper formalises, at the same place we do. The *half-slave* is the case where role-symmetry fails – he "can say 'I will set a price' but not 'you set a price"' (§2), so the priced clause cannot run – and he is also the case the court takes over and acts for on his own account. On the sugya's own terms the slave is freed on a separate rationale, not because he is a "ward"; the reading

that joins the two failures – of role-symmetry and into court guardianship – in one case is ours. With that caveat the coincidence is suggestive: ordinary co-owned property goes to *gud o agud*, the ward to the court-as-guardian, and the half-slave sits at the hinge, where Rav Naḥman's "the partnership continues" shades, for an object with standing, into the court's guardianship.

10 Conclusion

The buy–sell clause is a robust, belief-free device for dissolving co-ownership of an indivisible asset, and its Talmudic codification supplies design instruments – a valuation floor, a solvency requirement, a compel-versus-consent default – that map onto the clause's irreducible trilemma: one cannot dissolve efficiently, protect every owner's outside option, and allocate to the higher-valuer at once. When value is non-transferable the clause degenerates to allocation by wealth, or to nothing. And when the contested object has welfare of its own – a child, a holy site – the clause is not merely imperfect but mis-targeted: guardianship under a standard displaces it, and barring the priced channel is welfare-improving for the ward. The practical lesson for institutional design is a sorting rule: write a buy–sell clause for property, appoint a guardian for a ward, and treat the law's inalienability rules not as moralism but as the instrument that keeps the two apart.

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