Playing for the Winning Team:
Stepping Stones in Dynamic Talent Recruitment and Motivation

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Abstract. We study a dynamic coordination model of talent recruitment and motivation in which participation decisions endogenously affect the evolution of organizational status. In deterministic environments, strategic complementarities generate multiple equilibria: optimistic expectations attract talented recruits and so sustain winning teams, while pessimistic expectations lead to decline. The presence of noise in the evolution of status selects a unique equilibrium characterized by a ladder of participation thresholds across talent types. We identify a stepping-stone effect in which intermediate talent types lower the threshold required to attract elite recruits, even when such types are absent from realized equilibrium play in the vanishing-noise limit. This operates through two channels. Intermediate types smooth downside risk by slowing decline below the threshold ("downside smoothing") and compress participation thresholds so that elite types enter earlier ("upward triggering"). The same logic also governs effort provision, yielding an isomorphism between recruitment and incentives.

Talented individuals prefer to play for winning teams and so seek to join successful organizations. In this context, organizational success is self-reinforcing: a team that attracts a deep talent pool sustains growth in its status, while a team that fails to do so declines. Because the organization's future trajectory depends on who is willing to participate, recruitment decisions exhibit strategic complementarities. In deterministic environments, this generates multiple equilibria: optimistic expectations sustain success, while pessimistic expectations lead to decline.

We study a dynamic coordination environment in which participation decisions endogenously affect the drift of organizational status. The presence of noise in the evolving state variable selects a unique equilibrium in which each talent type requires the organization's status to exceed a threshold before participating: higher types contribute more to the organization but also possess better outside options, and therefore require higher thresholds. We characterize the resulting ladder of participation thresholds and obtain a particularly sharp characterization in the vanishing-noise limit.

Our analysis builds on the framework of Frankel and Pauzner (2000), who showed that introducing noise into environments with strategic complementarities selects a unique equilibrium.[2] We extend

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[2]Building upon the Matsuyama (1991) sectoral-choice model, Frankel and Pauzner studied returns to manufacturing that increase with participation and with a noisily evolving productivity variable. Their iterated-deletion procedure is related to infection arguments from the global-games literature (Carlsson and van Damme, 1993; Morris and Shin, 1998). Conceptually related step-by-step arguments appear in stochastic evolutionary dynamics of the kind proposed by Kandori, Mailath, and Rob (1993) and Young (1993), where intermediate states can influence long-run behavior (Ellison, 2000).

this framework by allowing participation decisions to endogenously affect the drift of the state process and by introducing heterogeneous outside options across participant types. These features generate a ladder of participation thresholds and a threshold-compression mechanism in which intermediate types alter equilibrium selection even when absent from realized equilibrium play in the vanishing-noise limit. In contrast to the original Frankel–Pauzner environment, the composition of available types affects equilibrium thresholds through its effect on endogenous state dynamics.[3]

A central finding is the identification of a stepping-stone effect: introducing an intermediate type lowers the threshold required for higher types to participate. This occurs even when such intermediate types do not directly determine long-run growth, in the sense that they are rarely pivotal participants. Nevertheless, their availability reshapes continuation values and alters equilibrium selection by strengthening the incentives of higher types to participate. In the vanishing-noise limit, intermediate types can affect equilibrium thresholds even when they are absent from realized equilibrium play. The composition of available types therefore influences organizational outcomes beyond the types that directly determine performance along the equilibrium path.

We further characterize how equilibrium thresholds respond to economic fundamentals. The ladder of participation thresholds decreases with the payoffs to participation and increases with participants' outside options. Equilibrium thresholds also depend asymmetrically on the volatility of the underlying state process: greater uncertainty in successful regimes raises the relevant thresholds, whereas volatility in failing regimes lowers them. This asymmetry reflects a dynamic survival logic: risk is costly when success is within reach but can be valuable when facing decline. Equilibrium selection therefore depends not only on expected performance but also on how uncertainty interacts with participation incentives and endogenous state dynamics.

We extend the framework to allow incumbent members to choose costly effort. In this environment, the expected success of the organization functions as an efficiency wage for members who face dismissal if they shirk. The effort environment exhibits the same threshold structure as the recruitment model, yielding an isomorphism between participation and incentive provision. This extends earlier work on dynamic political incentives with binary effort choices (Dewan and Myatt, 2012) by embedding effort incentives within the broader equilibrium-selection framework studied here. As in the recruitment setting, intermediate actions lower the threshold required to sustain high effort even when such actions are rarely used in equilibrium.

The mechanisms that we highlight arise in organizational settings where reputation or status shapes participation incentives and motivates team members. Universities, firms, political parties, and sports teams compete for participants whose outside options depend on perceived prospects. An academic department's ranking influences faculty and student recruitment; a sports club's trajectory affects player transfers; and a government's expected longevity shapes the willingness of skilled politicians to serve. In such environments, expectations and performance co-evolve through participation decisions that affect future organizational prospects. Our framework therefore relates naturally to work

[3]Applications of the Frankel–Pauzner framework include neighborhood choice (Frankel and Pauzner, 2002), debt runs and rollovers (He and Xiong, 2012; Cheng and Milbradt, 2012), and technology adoption (Guimaraes and Pereira, 2016; Crouzet, Gupta, and Mezzanotti, 2023). Our contribution differs by allowing endogenous drift through heterogeneous participation decisions and by identifying equilibrium-threshold effects driven by the structure of the type distribution.

on political selection and organizational performance. Caselli and Morelli (2004) showed how strategic complementarities in entry can generate multiple equilibria in which talented individuals refrain from participation, while our earlier work (Dewan and Myatt, 2010) examined the depletion of an exogenous talent pool within organizations. Here, by contrast, the talent pool is endogenous and participation decisions jointly determine equilibrium selection and state dynamics, yielding sharp comparative statics for how payoffs, outside options, and volatility shape participation thresholds.

We begin (Section 1) with a baseline single-type participation model and then extend (Section 2) to heterogeneous participant types and characterize the stepping-stone mechanism. We also apply the same logic to endogenous effort provision (Section 3) before offering concluding remarks (Section 4). Proofs and further discussion are contained in the appendix and online supplement.

1. A Simple Model of Evolving Status and Talent Recruitment

Our simple model of a recruiting organization (or "team") has two ingredients. Firstly, a team's evolving status depends upon whether talented recruits are available to it. Secondly, that availability is endogenous: potential recruits use the team's current status to predict career prospects within it.

Evolving Status and Recruitment. We study a many-player binary action game.

Players, Moves, and Payoffs. The players are a set of talented recruits indexed by $t \in [0, \infty)$. Recruit t takes a binary action $i_t \in \{0, 1\}$ at time t following the observation of the current status $x_t \in [0, \bar{x}]$ of an organization. This is a single "now or never" opportunity for the recruit, where $i = 1$ is a decision to join the team (or to be available to it) and $i = 0$ to taking up an outside option.

The team evolves in response to whether it is able to attract talented recruits ($i = 1$) or instead is limited either to lower-quality recruits or cannot recruit at all ($i = 0$). Specifically,

$$dx_t = \mu_i \, dt + \sigma_i \, dz_t \quad \text{where} \quad \mu_1 > 0 > \mu_0 \tag{1}$$

and where dz_t is the increment of a Wiener process. The state variable x_t is a reduced-form index of organizational status, which may be interpreted as reputation, performance, or perceived quality. When talented recruits are available ($i = 1$), status increases in expectation ($\mu_1 > 0$) as successful appointments offset attrition; otherwise ($i = 0$) status declines ($\mu_0 < 0$) as incumbents retire or performance deteriorates. The diffusion term captures small, continuous shocks to performance arising from factors beyond the control of any single recruit. We do not model explicitly the stock of team members; instead, the drift parameter summarizes the net expected effect of recruitment and attrition.

The team (and so game) continues in play until its status hits an upper or lower absorbing barrier. If $x_t = 0$ (the lower barrier) then we say that the team loses; if $x_t = \bar{x} > 0$ (the upper barrier) then the team wins. (The game ends at such barriers.)

Turning to payoffs, a talented recruit who joins the team enjoys a flow payoff u while the team continues in operation. A recruit discounts the future at rate ρ, and so $U \equiv u/\rho$ is the present value of a permanent career within a team that never reaches one of the two absorbing barriers. If the team ever wins then this changes to a terminal winning payoff W, but if it loses then the terminal payoff is L. We assume that $W > U > L$. The outside option is worth $\bar{U}$ which satisfies $W > \bar{U} > L$.

Optimality and Solution Concept. A strategy for the talented recruit at time t maps the observed status of the team to an "in or out" decision. Our players are Markovian: the decision is solely a function of the current team status. For a given (Markovian) strategy profile, we write $V(x)$ for the expected present value of a career within the team when the current team status is x.

A best reply for a recruit is to join the team if and only if the expected value of membership at least equals his outside option, so that $V(x) \geq \bar{U}$.[4] A value function $V(x)$ yields an equilibrium if the optimal participation decisions of talented recruits are consistent with that career value (so that $i_t = \mathcal{I}[V(x_t) \geq \bar{U}]$ where $\mathcal{I}[\cdot]$ is the indicator function) and if $V(x)$ arises from the strategy profile.

Modeling Choices and Interpretation. Formally, our players are talented recruits who arrive sequentially. They are atomistic and interact only through their collective effect on the organization's status. Equivalently, one may interpret the model as featuring a general supply of potential talent available at each moment of time, conditional on willingness to serve; this is our preferred interpretation. Each potential recruit faces a single "now or never" opportunity to participate in the talent pool. This captures environments in which talented individuals make career choices at discrete moments and do not strategically delay participation once an opportunity arises.

Focusing on "talented recruits" is equivalent to assuming two talent types: low-quality (or "bad") types who are always available and high-quality (or "good") types who endogenously decide whether to participate. The resulting talent pool therefore has two possible depths. In Section 2, we extend the analysis to a continuum of talent types and derive the corresponding ladder of participation thresholds.

The specification in (1) conditions the evolution of status solely on the depth of the talent pool. This abstracts from modeling explicitly the incumbent team composition or the stock of members. Allowing the drift or volatility to depend directly on current status can preserve the qualitative threshold structure.[5] The binary regime compactly captures the central feature that the organization flourishes if and only if sufficiently talented recruits are available.

Conditional on the depth of the talent pool, the diffusion term captures noise in the relationship between recruitment and performance. This may reflect imperfections in the recruitment process—an attempted hire of a high-quality recruit may fail, or a lower-quality recruit may prove unexpectedly successful—as well as other shocks to organizational outcomes. The diffusion also plays a conceptual role: it perturbs the deterministic coordination problem and selects a unique equilibrium.

The payoff structure, with terminal values satisfying $W > \max\{U, \bar{U}\} \geq \min\{U, \bar{U}\} > L$, provides a stark representation of the value of success relative to failure. We impose no restriction on the ordering of U and $\bar{U}$; recruits may value continued membership more or less than their outside option absent terminal outcomes. The absorbing barriers can be relaxed in special cases. For example, if $U > \bar{U}$, so that continued membership strictly dominates the outside option, the upper "winning" barrier may be removed by letting $\bar{x} \to \infty$.[6] The core threshold results extend to environments in which payoffs vary smoothly with status $u(x)$, in which case both barriers may be dispensed with.

[4]For example, a recruit that expects the team to play until T and then win participates if and only if $U + e^{-\rho T}(W - U) \geq \bar{U}$.
[5]For example, Propositions 1–2 continue to hold if μ_i is replaced by an increasing function $\mu_i(x)$.
[6]If instead $U < \bar{U}$, the lower absorbing barrier at zero may be eliminated.

A Deterministic Benchmark. As a benchmark, consider the noiseless case $\sigma_0 = \sigma_1 = 0$, in which the availability of talent perfectly determines the evolving fate of the organization.

The value $V(x)$ of team membership is strictly increasing in x. This means that recruits are willing to serve if and only if the team's status exceeds a threshold $x^\star$ that separates winning and losing teams.[7] Above it, the team's status increases for a period of time $(\bar{x} - x)/\mu_1$ until it reaches the winning line at $\bar{x}$. Crossing that line raises the value of a career from U (the present value of ongoing play) to W (a winner's payoff). Similarly, below the threshold a team declines until, after a period of time $x/|\mu_0|$, it experiences defeat (corresponding to a loss $U - L$) as a losing team. Incorporating discount rates,

$$V(x) = \begin{cases} U - e^{-\rho x/|\mu_0|}(U - L) & x < x^\star \\ U + e^{-\rho(\bar{x}-x)/\mu_1}(W - U) & x > x^\star \end{cases} \tag{2}$$

The threshold $x^\star$ generates an equilibrium if $V(x) \lesseqgtr \bar{U}$ for $x \lesseqgtr x^\star$.[8] There is an interval of values for $x^\star$ for which this is true. For sufficiently patient recruits, the entire interval $[0, \bar{x}]$ constitutes equilibria. Summarizing: any threshold within an appropriate range generates an equilibrium in which good types join the talent pool only if the team's status exceeds that threshold. Figure 1 illustrates.

Proposition 1 (Deterministically Evolving Status). *There is an interval $[x^\dagger, x^\ddagger] \subseteq [0, \bar{x}]$, such that any $x^\star \in [x^\dagger, x^\ddagger]$ yields an equilibrium in which: (i) the team attracts talented recruits and increases its status until it hits the winning line (it is a winning team) if $x > x^\star$; but (ii) it is unable to recruit successfully and it decays to failure (it is a losing team) if $x < x^\star$.*

The proof (contained, with other proofs, in the appendix) reports explicit solutions for $x^\dagger$ and $x^\ddagger$, and shows that $[x^\dagger, x^\ddagger]$ expands to fill the full interval $[0, \bar{x}]$ as ρ falls. Furthermore, this result also holds under a more general specification in which the flow payoff from team membership $u(x)$ and the drift rates of the team's status $\mu_0(x)$ and $\mu_1(x)$ are increasing functions of its current status.

There can be multiple self-fulfilling prophesies: winning teams ($x > x^\star$) that are expected to succeed attract the calibre required to win; lesser teams ($x < x^\star$) are expected to fail and do so. In the deterministic environment, multiplicity arises because expectations about future recruitment determine whether status rises or falls. However, the threshold $x^\star$ is not uniquely determined. This multiplicity problem is resolved fully when status evolves stochastically. Before doing this, however, we discuss briefly an informal argument for equilibrium selection.

A Selection Heuristic. For expositional simplicity set $\mu_1 = -\mu_0 = 1$ and $U = \bar{U} = 0$ (which means that the winning and losing terminal payoffs satisfy $W > 0 > L$) so that $V(x)$ simplifies to

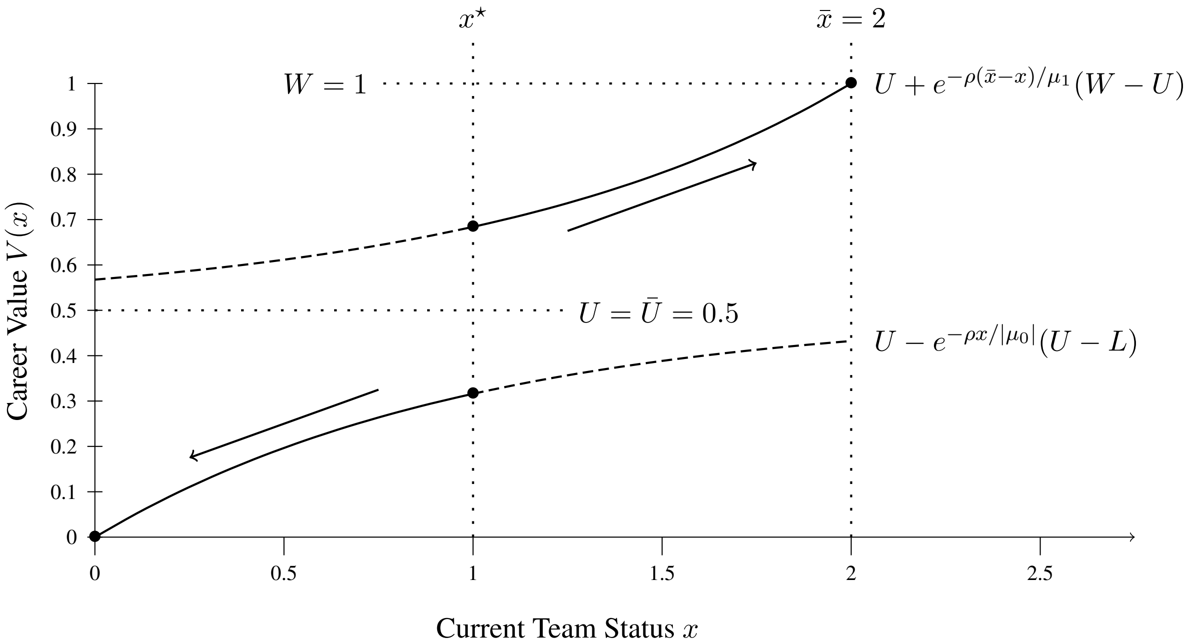
$$V(x) = \begin{cases} e^{-\rho x} L & x < x^\star \\ e^{-\rho(\bar{x}-x)} W & x > x^\star \end{cases} \tag{4}$$

[7] That threshold is $x^\star \equiv \inf\{x \in [0, \bar{x}] \mid V(x) \geq \bar{U}\}$. For now we put aside what happens when $x_t = x^\star$.

[8] This holds if and only if $\lim_{x \uparrow x^\star} V(x) \leq \bar{U} \leq \lim_{x \downarrow x^\star} V(x)$, or equivalently if

$$e^{-\rho(\bar{x}-x^\star)/|\mu_1|}(W - U) \geq \bar{U} - U \geq -e^{-\rho x^\star/|\mu_0|}(U - L). \tag{3}$$

One of these inequalities is always satisfied. If $U > \bar{U}$ (a lifetime in play is better than the outside option) then the first inequality holds; there is no lower bound to $x^\star$. If instead $U < \bar{U}$ (a talented recruit only joins if motivated by the winner's payoff W) then the second inequality holds; there is no upper bound to $x^\star$. In each case, the other inequality places some restriction on $x^\star$. However, both inequalities are satisfied for all $x^\star \in [0, \bar{x}]$ if recruits are patient (so that ρ is small).



Notes. This illustrates one of many equilibria when status evolves deterministically. We set the payoff parameters to $L = 0$, $U = \bar{U} = 0.5$, and $W = 1$. (These choices mean that a recruit joins a team if he thinks it is more likely to win rather than lose.) Our other parameter choices are $\mu_1 = 1$, $\mu_0 = -1$, $\rho = 1$, and $\bar{x} = 2$. The top upward sloping path illustrates the career value when a winning team attracts talented recruits; the lower downward path illustrates the career value in a losing team. We have illustrated an equilibrium threshold $x^\star = 1$. At this point the outside-option value of a talented recruit lies in between the two aforementioned paths. In fact, for these parameters, any $x^\star \in [0, \bar{x}]$ generates an equilibrium.

FIGURE 1. An Equilibrium with Deterministically Evolving Status

Consider a recruit observing a team with status close to the threshold $x \approx x^\star$. With small stochastic shocks to status, the team may move into either regime in the next instant. The career values on the two sides of the threshold are $\lim_{x \downarrow x^\star} V(x) = e^{-\rho(\bar{x}-x^\star)}W$ and $\lim_{x \uparrow x^\star} V(x) = e^{-\rho x^\star} L$ respectively. If positive and negative shocks are locally symmetric, then when $x = x^\star$ the recruit anticipates equal probabilities of movement toward winning or losing trajectories. Indifference therefore requires

$$0 = \frac{e^{-\rho x^\star} L + e^{-\rho(\bar{x}-x^\star)} W}{2} \quad \Longrightarrow \quad x^\star = \frac{1}{2}\left(\bar{x} - \frac{1}{\rho}\log\left[\frac{W}{|L|}\right]\right). \tag{5}$$

This threshold increases with the distance ($\bar{x}$) to the winning line and decreases with the gain-to-pain ratio from winning rather than losing ($W/|L|$). A recruit's concern that the team may experience a small shock when close to the threshold ties together the winning and losing career valuations from either side. The equilibrium illustrated in Figure 1 satisfies this heuristic criterion.

This reasoning mirrors the equilibrium-selection logic of global games (Carlsson and van Damme, 1993; Morris and Shin, 1998) in which threshold strategies arise from local indifference under symmetric uncertainty following the "Laplacian beliefs" logic of Morris and Shin (2003, pp. 62–64). Here, stochastic evolution in team status plays the role of perturbing the deterministic coordination problem. Whereas the argument here is heuristic, the conjectured solution reported in eq. (5) emerges from the analysis of an environment with stochastically evolving team status, as we now show.

Noisily Evolving Status. We now return to the full model in which $\sigma_i > 0$ for each $i \in \{0, 1\}$.

As before, a threshold $x^\star$ separates winning and losing teams. The key difference is that $V(x)$ is now continuous. In the absence of noise $V(x)$ experiences a jump upward at the threshold $x^\star$. With noise, however, it is a continuously differentiable (that is, C^1) function: the presence of random shocks smooths out the gap between optimistic (winning team) and pessimistic (losing team) worlds.

Lemma 1 (Career Values)**.** *Fixing the (Markovian) behavior of others, the career value function $V(x)$ is strictly increasing and continuously differentiable. It satisfies $V(0) = L$ and $V(\bar{x}) = W$. There is a threshold $x^\star \in (0, \bar{x})$ satisfying $V(x^\star) = \bar{U}$ and so talented recruits join if and only if $x \geq x^\star$.*

The formal proof (in the appendix) applies work by Strulovici and Szydlowski (2015) concerning the smoothness of value functions in diffusion models. Informally, just below the threshold a recruit recognizes that the noise in evolving status can move the team above the threshold, and vice versa. Over short intervals, noise dominates drift, and this ensures the continuity of $V(x)$.

Next, we establish the existence of a unique equilibrium threshold. To do this, we extend our notation to write $V(x \,|\, x^\star)$ for the career value when the team's status is x and other talented recruits use the threshold $x^\star$. This is continuous in its arguments and has the properties (with respect to x) described in Lemma 1. We also define $\bar{V}(x) = V(x \,|\, x)$ as the career value when the current team status x is equal to the threshold used by others: $x^\star$ is an equilibrium threshold if and only if $\bar{V}(x^\star) = \bar{U}$.

Existence and uniqueness now follow because $\bar{V}(x)$ is continuously increasing from $\bar{V}(0) = L < \bar{U}$ up to $\bar{V}(\bar{x}) = W > \bar{U}$. To see why, consider the evolution of status relative to the threshold. The statistical properties of this are independent of the threshold itself: it satisfies eq. (1) where $i = 1$ above the threshold, but $i = 0$ below it. For a team member standing at the threshold, the evolving prospects of the team (again judged relative to that threshold) look the same. However, a higher threshold is strictly closer to the winning line (and the prize $W > U$) and strictly further from the losing line (and a losing terminal payoff $L < U$). Thus, the career value beginning from that threshold is strictly higher. We conclude that $\bar{V}(x)$ is increasing in x. Together with continuity, this implies that there is a unique $x^\star$ satisfying $\bar{V}(x^\star) = \bar{U}$.

The argument sketched here fixes the flow payoff u enjoyed by a team member. If we vary this assumption and allow the flow payoff to be $u(x)$, an increasing function of x, then the same argument can be used: beginning from a higher threshold, a team member expects the same statistical properties of evolving team status relative to that threshold but experiences higher flow payoffs from membership of the team. We can also allow the properties of the team's evolving status to depend on its current status. If $\mu_1(x)$ and $\mu_0(x)$ are both increasing functions of x then the argument also holds: starting from a higher threshold, a team member expects higher drift upwards (higher $\mu_1(x)$ above the threshold) or lower drift downwards (higher $\mu_0(x)$ below the threshold, or, equivalently, lower $|\mu_0(x)|$) which enhances the career value.

The same logic underpins comparative-static exercises. For example, an increase in any team payoff parameter (W, u, and L) raises $\bar{V}(x)$ and lowers the threshold $x^\star$ at which it intersects $\bar{U}$.

These claims (existence, uniqueness, and comparative statics) are reported formally in Proposition 2.

Proposition 2 (Equilibrium)**.** *There is a unique equilibrium with a threshold $x^\star$ such that talented recruits are willing to play for the team if and only if its status is at least $x^\star$.*

This claim also holds if the flow payoff u from team membership or if the drift rates μ_0 and μ_1 of eq. (1) *depend positively on the status of the team.*

The equilibrium threshold $x^\star$ is decreasing (this is helpful for the team's success) in the payoffs W (the prize for winning), u (the flow payoff from playing), and L (the terminal payoff from losing) associated with team membership; it is increasing in the outside option $\bar{U}$ of a talented recruit.

The threshold $x^\star$ is also decreasing in the drift rates of status in both winning and losing teams.

Finally, the equilibrium threshold $x^\star$ is increasing in $\bar{x}$, which is the distance to the winning line.

The arguments described above, and which underpin the proof of Proposition 2, are closely related to those used by Frankel and Pauzner (2000), and so we describe the relationship here.

The argument underlying Proposition 2 parallels that of Frankel and Pauzner (2000), who showed that introducing noise into environments with strategic complementarities selects a unique equilibrium. As in their setting, the relative dynamics beginning from a point of indifference are invariant to the level of the threshold, which yields uniqueness. Our environment differs in allowing endogenous drift through recruitment decisions and in emphasizing comparative statics of the threshold.

Vanishing Noise. Propositions 1 and 2 extend to a richer model in which the flow payoffs and drift rates depend positively on status. Here we restrict to the constant-parameters case in order to obtain closed-form expressions and to characterize sharply how volatility affects the equilibrium threshold.

We begin by reporting an explicit solution for the career value function.

Lemma 2. *On either side of the equilibrium threshold $x^\star$, the career value function satisfies:*

$$V(x) = U + \begin{cases} (\bar{U}-U)\frac{e^{-b_0^- x}-e^{-b_0^+ x}}{e^{-b_0^- x^\star}-e^{-b_0^+ x^\star}} + (L-U)\frac{e^{b_0^+(x^\star-x)}-e^{b_0^-(x^\star-x)}}{e^{b_0^+ x^\star}-e^{b_0^- x^\star}} & x < x^\star \\ (W-U)\frac{e^{-b_1^-(x-x^\star)}-e^{-b_1^+(x-x^\star)}}{e^{-b_1^-(\bar{x}-x^\star)}-e^{-b_1^+(\bar{x}-x^\star)}} + (\bar{U}-U)\frac{e^{b_1^+(\bar{x}-x)}-e^{b_1^-(\bar{x}-x)}}{e^{b_1^+(\bar{x}-x^\star)}-e^{b_1^-(\bar{x}-x^\star)}} & x > x^\star \end{cases}, \tag{6}$$

where the various "b" coefficients are $b_i^\pm \equiv \left[\mu_i \pm \sqrt{\mu_i^2 + 2\rho\sigma_i^2}\right]/\sigma_i^2$ for each $i \in \{0,1\}$.

Lemma 2 characterizes $V(x)$ conditional on the threshold $x^\star$. Given that this career-value function is continuously differentiable under $\sigma_i > 0$ (from Lemma 1), the left and right derivatives must coincide at $x^\star$. Differentiating on either side and (without loss of generality) normalizing $U = 0$ (so that other payoffs are measured relative to U; this does not alter $x^\star$) the smooth-pasting condition becomes

$$\frac{\bar{U}(b_0^+ e^{b_0^- x^\star} - b_0^- e^{b_0^+ x^\star}) - L(b_0^+ - b_0^-)}{e^{b_0^+ x^\star} - e^{b_0^- x^\star}} = \frac{W(b_1^+ - b_1^-) - \bar{U}(b_1^+ e^{-b_1^-(\bar{x}-x^\star)} - b_1^- e^{-b_1^+(\bar{x}-x^\star)})}{e^{-b_1^-(\bar{x}-x^\star)} - e^{-b_1^+(\bar{x}-x^\star)}}. \tag{7}$$

Finding the threshold $x^\star$ that solves this equation pins down completely the equilibrium.[9]

The primitives enter through the coefficients $b_i^\pm$, but the equilibrium condition simplifies in the small-noise limit. To proceed here, we use a scaling parameter ξ to control the volatility in the evolution of

[9]The proof of Proposition 3 shows, for completeness, that there is a unique solution; this reinforces Proposition 2.

the team's status. Specifically, we now modify eq. (1) so that it becomes

$$dx_t = \mu_i\, dt + \xi\sigma_i\, dz_t. \tag{8}$$

where $\xi > 0$ scales the magnitude of shocks, holding fixed their relative variances σ_1^2/σ_0^2. Allowing ξ to fall toward zero, the various coefficients $b_i^{\pm}$ simplify appreciably.[10]

Proposition 3 (Vanishing Noise)**.** *Suppose that the team's status evolves via $dx_t = \mu_i\, dt + \xi\sigma_i\, dz_t$. The limit of the equilibrium threshold $x^\diamond = \lim_{\xi\to 0} x^\star$ exists and is unique. It satisfies*

$$\frac{|\mu_0|((U-L)e^{-\rho x^\diamond/|\mu_0|} - (U-\bar{U}))}{\sigma_0^2} = \frac{\mu_1((W-U)e^{-\rho(\bar{x}-x^\diamond)/\mu_1} + (U-\bar{U}))}{\sigma_1^2}, \tag{9}$$

if the limit satisfies $x^\diamond \in (0,\bar{x})$. Otherwise the limit is a corner: $\lim_{\xi\to 0} x^\star \in \{0,\bar{x}\}$.

This limit $x^\diamond$ of the equilibrium threshold that satisfies eq. (9)

(i) is decreasing in the payoffs (W, u, L) of team membership, but increasing in the outside option $\bar{U}$;

(ii) is decreasing in the rate of increase μ_1 in a winning team (when the team can recruit talent), but increasing in the magnitude of the rate of decline $|\mu_0|$ in a losing team (when it cannot);

(iii) is increasing in $\bar{x}$, which measures the distance to the winning line; and

(iv) is increasing in the relative volatility σ_1^2/σ_0^2 in winning versus losing teams.

Claims (i), (ii), and (iii) all reprise the comparative-static results described in Proposition 2.

Claim (iv) highlights the asymmetric role of volatility. Greater noise in winning states (an increase in σ_1^2) raises the equilibrium threshold and therefore makes success harder to sustain, whereas greater noise in losing states (an increase in σ_0^2) lowers the threshold and can facilitate recovery.

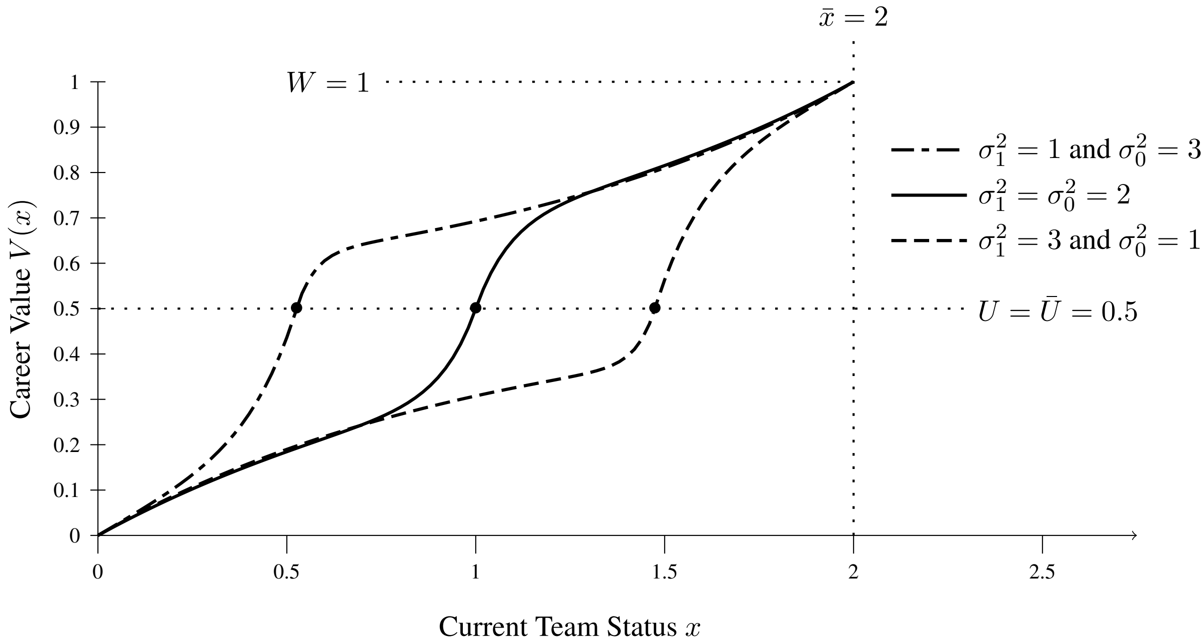
In the absence of noise, a team above the threshold reaches the winning barrier deterministically, so additional uncertainty is harmful. Below the threshold, however, the team is destined to fail absent shocks; volatility introduces the possibility of crossing into the winning regime. Thus, a winning team benefits from stability, whereas a losing team may benefit from risk—a "gamble for resurrection." This asymmetry echoes the logic of bold play in survival problems (Dubins and Savage, 1965; Dutta, 1994). Here, the status of the organization plays the role of wealth, and the preferred degree of risk depends on proximity to the threshold separating success from failure.

Importantly, volatility (and in particular the relative importance of volatility in the winning versus losing regimes) affects equilibrium outcomes even when it appears negligible for continuation values away from the threshold. Indeed, the limiting solution for the career value function is

$$\lim_{\xi\to 0} V(x) = \begin{cases} U - e^{-\rho x/|\mu_0|}(U-L) & x < x^\diamond \\ U + e^{-\rho(\bar{x}-x)/\mu_1}(W-U) & x > x^\diamond \end{cases} \tag{10}$$

so that career values away from $x^\diamond$ are independent of σ_0^2 and σ_1^2 in the vanishing-noise limit. Volatility (here, this is recruitment risk) matters precisely because it pins down the equilibrium threshold itself.

[10]These simplifications are $\lim_{\xi\to 0}\mu_1 b_1^- = \lim_{\xi\to 0}\mu_0 b_0^+ = -\rho$ and $\lim_{\xi\to 0}\sigma_1^2 b_1^+/\mu_1 = \lim_{\xi\to 0}\sigma_0^2 b_0^-/\mu_0 = 2$. These properties also imply that $\lim_{\xi\to 0} b_1^+ = \infty$ and $\lim_{\xi\to 0} b_0^- = -\infty$. This ensures that terms in eq. (7) involving $e^{-b_1^+(\bar{x}-x^\star)}$ and $e^{b_0^- x^\star}$ do not matter in the limit, and so eq. (7) simplifies appreciably.



Notes. This illustrates equilibria for different relative choices of σ_i^2. We have maintained the parameter choices used in Figure 1: payoff parameters are $L = 0$, $U = \bar{U} = 0.5$, $W = 1$, $\mu_1 = 1$, $\mu_0 = -1$, $\rho = 1$, $\bar{x} = 2$, and $\xi^2 = 0.1$. We compute the unique equilibrium threshold $x^\star$ and the corresponding career value function for three different configurations of σ_0^2 and σ_1^2.

FIGURE 2. Equilibria with Stochastically Evolving Status

The effect of relative noise is illustrated in Figure 2. Building upon the specification that we used in Figure 1, we have added noise and so pinned down (for each parameter configuration) the unique equilibrium. Note that as evolving status in the winning-team world becomes relatively riskier, we move from left to right: the equilibrium threshold increases noticeably.

We now describe explicit solutions for two special cases. We recall that recruits care about the value of ongoing play (via U) as well as the final outcome when a team wins or loses (via W and L).

Proposition 4 (Special Cases)**.** *Consider the limit of the unique equilibrium threshold $x^\diamond = \lim_{\xi \to 0} x^\star$.*

(i) If talented recruits care only about the long-run survival of their team, so that $W = U$, then

$$x^\diamond = \frac{|\mu_0|}{\rho}\left[\log\left(\frac{U-L}{U-\bar{U}}\right) + \log\left(\frac{|\mu_0|/\sigma_0^2}{(|\mu_0|/\sigma_0^2) + (\mu_1/\sigma_1^2)}\right)\right]. \tag{11}$$

(ii) If talented recruits care only about winning and losing, so that $U = \bar{U} = 0$, then

$$x^\diamond = \frac{|\mu_0|}{\mu_1 + |\mu_0|}\left(\bar{x} + \frac{\mu_1}{\rho}\left[\log\left(\frac{|L|}{W}\right) + \log\left(\frac{|\mu_0|/\sigma_0^2}{\mu_1/\sigma_1^2}\right)\right]\right). \tag{12}$$

In the survival-only case (i), the threshold increases with patience: when ρ is small, the risk of eventual failure weighs more heavily and so recruits require a higher status before joining. If the recruit is impatient then the prospect of a losing team is less pressing, which raises the incentive to join.

The win-loss-only case (ii) is when a recruit cares only about the prize from winning or the pain from losing. Under symmetric drift and volatility ($\mu_1 = -\mu_0 = 1$ and $\sigma_0^2 = \sigma_1^2$) this solution (for parameters under which an interior solution applies) reduces to

$$x^{\diamond} = \frac{1}{2}\left(\bar{x} - \frac{1}{\rho}\log\left[\frac{W}{|L|}\right]\right), \tag{13}$$

which matches the heuristic discussion following Proposition 1. The limiting threshold is decreasing in the gain-to-pain ratio of the payoffs W (the prize when the team wins) and $|L|$ (the pain when it loses). The effect of changing patience also depends on whether the prize exceeds the pain.[11]

Summary. The baseline model illustrates how stochastic evolution of organizational status selects a unique participation threshold in an environment with strategic complementarities. The vanishing-noise analysis highlights an asymmetric role for volatility: stability benefits organizations above the threshold, whereas risk can assist those below it. These results apply broadly to settings in which expectations about future success shape present recruitment decisions, including political, academic, and sporting organizations. We now extend the analysis to heterogeneous talent and examine how the distribution of potential types reshapes equilibrium thresholds.

2. Stepping Stones in the Talent Pool: A Model with Multiple Types

We now allow for multiple levels of talent. Our central finding is that the presence of intermediate types lowers the threshold required to attract higher performers, even when such types are never recruited along the equilibrium path in the deterministic limit.

Multiple Types of Recruit. We allow for n recruit types indexed by $i \in \{1, \ldots, n\}$, where a higher index indicates greater talent. We set $i = 0$ to denote the absence of recruitment.

Players, Moves, and Payoffs. The players are recruits of n different talent levels indexed by $(i, t) \in \{1, \ldots, n\} \times [0, \infty)$. At time t there are n recruit types (corresponding to the n different talent indices) who make now-or-never binary-action decisions to make themselves available for recruitment by participating in a team's talent pool. We write $i_t \in \{0, \ldots, n\}$ for the highest talent level in the pool (in essence, its depth) at time t, with $i_t = 0$ if no type is willing to serve.

Status evolves according to $dx_t = \mu_i \, dt + \sigma \, dz_t$ when the best available recruit is type i. We assume homoscedastic noise, so that σ does not depend on i, and label types so that $\mu_0 < \mu_1 < \cdots < \mu_n$. This set of types is partitioned into good and bad: there is some $i^{\circ} \in \{1, \ldots, n\}$ such that $\mu_{i^{\circ}-1} < 0 < \mu_{i^{\circ}}$, and so the team's status grows in expectation if and only if the talent pool is sufficiently deep. Many of our results hold with a less restrictive specification.[12]

A recruit into the team from the talent pool enjoys the same payoffs as before. Those with more talent enjoy better outside opportunities: we assume that $L < \bar{U}_1 < \cdots < \bar{U}_n < W$ where $\bar{U}_i$ is the outside option for type i. For convenience of notation we set $\bar{U}_0 \equiv L$ and $\bar{U}_{n+1} \equiv W$.

[11] With symmetric drift and volatility terms and with appropriately patient recruits (ρ is sufficiently small) the team always wins ($\lim_{\xi \to 0} x^{\star} = 0$) if $W > |L|$ but always loses ($\lim_{\xi \to 0} x^{\star} = \bar{x}$) if $|L| > W$.

[12] Specifically, we can (sometimes with suitable parameter restrictions) accommodate heteroscedastic specifications in which σ_i depends on the type of recruit. Furthermore, we do not need to impose monotonicity of μ_i: for many results we can use a weaker single-crossing property, so that there is some $i^{\circ} \in \{1, \ldots, n\}$ such that $\mu_i > 0$ if and only if $i \geq i^{\circ}$. Of course, we can also allow (just as before) drift rates and flow payoffs to depend (monotonically) on the state of play.

Optimality and Solution Concept. A strategy for type i at time t maps the observed status x_t to a binary participation decision. Our players remain Markovian, and $V(x)$ is the value of a career within a team of status x. A type i joins the talent pool if and only if $V(x) \geq \bar{U}_i$. In particular, attracting the lowest good type i° requires $V(x) \geq \bar{U}_{i^\circ}$. An equilibrium follows the same definition as before.

The Deterministic Benchmark. We again consider the deterministic benchmark with $\sigma = 0$. As before, $V(x)$ is increasing, so there exists a threshold $x^\star$ such that $V(x) \gtrless \bar{U}_{i^\circ}$ for $x \gtrless x^\star$, separating winning and losing regions. There is also a specific threshold $x_i^\star$ for each recruit type which induces that type's participation: $V(x) \gtrless \bar{U}_i$ for $x \gtrless x_i^\star$. These type-specific thresholds may coincide. Whether they coincide or not, these thresholds must satisfy $x_1^\star \leq x_2^\star \leq \cdots \leq x_n^\star$.

For the lower range of status $x < x^\star$, the team cannot attract good types and its status declines. Types with $\bar{U}_i > U$ never join a losing team, while types with $\bar{U}_i < U$ may participate if the remaining lifetime is sufficiently long. For example, if $U > \bar{U}_1$, type 1 joins whenever $x > \bar{x}_1$, where

$$\bar{x}_1 = \frac{|\mu_0|}{\rho} \log\left(\frac{U - L}{U - \bar{U}_1}\right). \tag{14}$$

As the team's status falls, only progressively lower types remain in the talent pool, and so the losing team's decline accelerates as the talent pool evaporates.

Analogously, for the higher range of status $x > x^\star$, higher types require greater proximity to the winning barrier. For example, if $U < \bar{U}_n$, then type n (the elite type) joins only when $x > \bar{x}_n$, where

$$\bar{x}_n = \bar{x} - \frac{\mu_n}{\rho} \log\left(\frac{W - U}{\bar{U}_n - U}\right). \tag{15}$$

As status rises, increasingly talented recruits enter the talent pool of this winning team, accelerating convergence to the winning line. For both winning and losing teams we can fully characterize the thresholds at which the talent pool expands and contracts. These thresholds are reported in the next proposition, which generalizes Proposition 1 to the presence of multiple types.[13]

Proposition 5 (Equilibrium with Deterministic Quality and Many Types)**.**

(i) Proposition 1 holds with multiple types: there is an interval $[x^\dagger, x^\ddagger]$ such that any $x^\star \in [x^\dagger, x^\ddagger]$ yields an equilibrium with a winning team for $x > x^\star$, and a losing team for $x < x^\star$.

(ii) For $i \leq i^\circ$, corresponding to the lowest types, define $\bar{x}_0 \equiv 0$ and then iteratively

$$\bar{x}_i = \begin{cases} \min\left\{\bar{x}_{i-1} + \frac{|\mu_{i-1}|}{\rho}\log\left(\frac{U - \bar{U}_{i-1}}{U - \bar{U}_i}\right), \bar{x}\right\} & \bar{U}_i < U \\ \bar{x} & \bar{U}_i \geq U \end{cases} \tag{16}$$

The critical status threshold that induces the participation of type $i < i^\circ$ is $x_i^\star = \min\{x^\star, \bar{x}_i\}$.

(iii) For $i \geq i^\circ$, corresponding to the highest types, define $\hat{x}_{n+1} \equiv \bar{x}$ and then iteratively

$$\hat{x}_i = \begin{cases} \max\left\{\hat{x}_{i+1} - \frac{\mu_i}{\rho}\log\left(\frac{\bar{U}_{i+1} - U}{\bar{U}_i - U}\right), 0\right\} & \bar{U}_i > U \\ 0 & \bar{U}_i \leq U \end{cases} \tag{17}$$

The critical status threshold that induces the participation of type $i > i^\circ$ is $x_i^\star = \max\{x^\star, \hat{x}_i\}$.

[13] We can also characterize readily career values in this equilibrium. If $x_i^\star < x < x_{i+1}^\star$ where $i < i^\circ$ then $V(x) = U - e^{-\rho(x - x_i^\star)/|\mu_i|}(U - \bar{U}_i)$. Similarly, if $i \geq i^\circ$ then $V(x) = U + e^{-\rho(x_{i+1}^\star - x)/|\mu_i|}(\bar{U}_{i+1} - U)$.

As in the binary case, $x^\star$ is not uniquely determined in the deterministic environment. Furthermore, if players are sufficiently patient then the interval of possible equilibrium thresholds expands to $[0, \bar{x}]$.

If the team lies within the "win zone" then increased status brings in progressively higher types. However, there are situations in which all good types participate once the threshold is crossed.

One such situation is when perpetual play is enough to tempt all good types into the talent pool. Specifically, if $U > \bar{U}_n$, so that perpetual play is sufficiently valuable, participation of the lowest good type i° guarantees survival and hence $V(x) \geq U$. This is then enough to bring in all other types: the entry of one good type acts as a "stepping stone" to recruit all others. On the other hand, if time in play is not sufficiently valued then a fall into the losing-team zone will scare away all possible recruits. Both of these special cases are reported in the following corollary.

Corollary (to Proposition 5)**.**

(i) If $U > \bar{U}_n$, so that perpetual play is valuable, then $x_i^\star = x^\star$ for all $i \geq i^\circ$: if the team can recruit the lowest good type recruit, then all other good types, including the highest, join the talent pool.

(ii) If $U < \bar{U}_1$, so that the prospect of winning is needed to induce participation, then $x_i^\star = x^\star$ for all $i \leq i^\circ$: a losing team is unable to attract any positive-type recruit into the talent pool.

Case (i) is important. It describes a situation in which any winning-team equilibrium necessarily entails participation of the elite type n (since $V(x) \geq U > \bar{U}_n$ whenever the team is in the winning region). Consequently, any intermediate good type $i \in \{i^\circ, \dots, n-1\}$ is never a marginal member of the talent pool and is never recruited along the equilibrium path. This might suggest that such types are irrelevant. We will show that they are not once status evolves stochastically: even types that are never recruited along the equilibrium path of a deterministic equilibrium can reshape the equilibrium threshold through their mere availability.

Noisily Evolving Status. We now allow for noise in the evolution of the team, so that $\sigma > 0$.

Lemma 1 applies here: fixing the (Markovian) behavior of others, the value of a career smoothly and strictly increases from L to W over the range of play. This implies the existence of a threshold $x_i^\star$ for each type $i \in \{1, \dots, n\}$ at which that career value within the team matches the corresponding outside option $\bar{U}_i$. These thresholds are strictly ordered, $0 < x_1^\star < \cdots < x_n^\star < \bar{x}$, reflecting the strict ordering of outside options $\bar{U}_1 < \cdots < \bar{U}_n$. We assemble them into a vector $\mathbf{x}^\star \in X$ where $X = \{\mathbf{x} \in [0, \bar{x}]^n : x_1^\star \leq \cdots \leq x_n^\star\}$. Extending earlier notation, we write $V(x \,|\, \mathbf{x}^\star)$ for the career value of a team member when the team's current status is x and when other recruit types use the thresholds $\mathbf{x}^\star \in X$. This is strictly increasing in x, and strictly decreasing in each threshold $x_i^\star$.[14] Clearly, $\mathbf{x}^\star \in X$ corresponds to an equilibrium if $\bar{U}_i = V(x_i^\star \,|\, \mathbf{x}^\star)$ for all i.

To show existence, we construct a sequence of best replies in the space X. Indexing by $s \in \{0, 1, \dots\}$, we begin with $\mathbf{x}^{(0)}$ defined by $x_i^{(0)} = \bar{x}$ for all i, corresponding to the profile in which no recruit joins. For $s \in \{1, 2, \dots\}$ define iteratively $x_i^{(s)}$ to be the optimal threshold used by a type i given that others use the relevant threshold from $\mathbf{x}^{(s-1)} \in X$. That is, choose $\mathbf{x}^{(s)}$ to satisfy $\bar{U}_i = V(x_i^s \,|\, x^{(s-1)})$ for each i. (Any threshold exceeding $x_i^{(1)}$ is a dominated strategy for type i.) Continuing iteratively, this

[14]Lowering the threshold used by any recruit type accelerates the drift upwards of team status toward higher payoffs, and so increases the present value of a career in the team.

means that only thresholds that fall below $x_i^{(s)}$ survive s rounds of iterative deletion. This constructs an infection argument in the style of the global games literature, mirroring Frankel and Pauzner (2000). Continuing in this manner, the strictly decreasing sequence $\{\mathbf{x}^{(s)}\}$ converges to $\mathbf{x}^\star \equiv \lim_{s\to\infty} \mathbf{x}^{(s)}$. The continuity of the career value function ensures that $V(x_i^\star \,|\, \mathbf{x}^\star) = \bar{U}_i$, and so $\mathbf{x}^\star$ is an equilibrium.

We could also perform a similar procedure beginning from the most optimistic strategy profile in which all recruits are always willing to join the team. The proof of the next proposition confirms that this procedure converges to the same limit, and there is a unique equilibrium.[15]

Proposition 6 (Equilibrium with Stochastically Evolving Status and Many Types)**.** *In a multiple-type world with noisily evolving status, there is a unique set of thresholds satisfying $0 < x_1^\star < x_2^\star < \cdots < x_n^\star < \bar{x}$ such that type i is willing to play for the team if and only if its status is at least $x_i^\star$.*

It remains to obtain a sharper characterization of this equilibrium. We do this, as before, by considering the properties of the equilibrium thresholds as the noise in the system vanishes, so that $\sigma^2 \to 0$.

Stepping Stones in the Talent Pool. With a view to streamlining our exposition, we now consider an environment in which the dominant motivation for a participating team player is the desire to avoid the team's defeat. To do this, we set $U = W$ and allow $\bar{x} \to \infty$.[16] Since perpetual play is equivalent to victory, letting $\bar{x} \to \infty$ removes the upper absorbing barrier and yields a simpler characterization.

Since $U = W$, we have $U > \bar{U}_i$ for all i, including the elite type n. In a noiseless world (that is, when $\sigma^2 = 0$) this means that if recruits expect the team's status to increase then the value of a career in the team is U, which attracts all good types of recruit. Thus, recruitment of the lowest good type i° implies recruitment of the elite type n. A corollary is that intermediate good types satisfying $i \in \{i^\circ, i^\circ + 1, \ldots, n-1\}$ are never actively recruited: once they become available, all better types immediately follow and so a team can cherry pick the very best.

We record these observations as a corollary to Proposition 5.

Corollary (to Proposition 5)**.** *For $U = W$ consider an equilibrium with deterministically evolving status in which there is a threshold that determines whether the team is on a winning or losing path. The individual participation thresholds of all good types coincide with that threshold.*

This suggests that mediocre recruits (good types that are not the very best) have little impact. This is true in the sense that the addition or deletion of them from the set of possible recruits makes no significant difference to the set of equilibria in a noiseless world. However, and as we show, their presence does influence the unique equilibrium when status evolves stochastically. Furthermore, taking noise out of the system selects a unique equilibrium from those in Proposition 5. That selection

[15] The proof of Proposition 6 also uses the logic from the earlier Proposition 2: for two sets of similarly spaced thresholds, the team's evolving status follows the same law of motion relative to those thresholds. The higher set of thresholds is closer to the winning line and higher payoffs, and so must generate a higher career value when the team's current status is evaluated at one of those thresholds. This argument is used to eliminate multiple equilibria.

[16] These parameter choices (which correspond to the assumptions that there is no winning line and that recruits value care about the team's longevity) allow us to express our key expressions compactly. Our key formal results also hold if, for example, $W > U > \bar{U}_n$ so that the expectation of long-run team survival is enough to induce the participation of the elite type n. We can readily allow for $\bar{U}_n > U$ at the cost of substantial clutter in the statement of our results.

also depends on the presence of mediocre types, even though the equilibrium selected does not involve their successful recruitment to the team. (They are hidden or "underwater" stepping stones.)

To make this claim formally, we begin by stating some basic properties of the equilibrium thresholds as noise vanishes from the system. This next lemma shows that the status thresholds of all good types (and possible some bad types too) all collapse to a single point in the limit.

Lemma 3 (Equilibrium Properties). *Suppose that status evolves via* $dx_t = \mu_i\, dt + \sigma\, dz_t$. *There is a unique* $x^\diamond$ *such that* $\lim_{\sigma\to 0} x_i^\star = x^\diamond$ *for all* $i \geq i^\circ$*: in the limit, as noise vanishes, all good types use the same threshold of team status to determine their participation in the talent pool. For* $i < i^\circ$*, the equilibrium thresholds of bad types satisfy* $\lim_{\sigma\to 0} x_i^\star = \min\{x^\diamond, \bar{x}_i\}$*, where* $\bar{x}_i$ *is from Proposition 5.*

Just as in the two-type case, the elimination of noise selects a threshold $x^\diamond$ that partitions winning and losing teams. An implication of this lemma is that there exists a pivotal type $i^\ddagger \leq i^\circ$ such that $x_i^\star \to x^\diamond$ if and only if $i \geq i^\ddagger$. If a team's status is strong enough to bring in this pivotal type $i^\ddagger$ (where we note that this may actually be a bad type if $i^\ddagger < i^\circ$) then all other higher types (including the highest or elite type n) follow into the talent pool. Our next lemma characterizes the identity of this pivotal recruit type whose recruitment prompts the participation of all others.

For this lemma, recall that we defined $\bar{U}_0 = L$ and $\bar{U}_{n+1} = W$.

Lemma 4 (Pivotal Recruit Type). *If* $\sum_{j=0}^{i^\circ-1} |\mu_j|(\bar{U}_{j+1}-\bar{U}_j) \leq \sum_{i^\circ}^{n} \mu_j(\bar{U}_{j+1}-\bar{U}_j)$ *then* $\lim_{\sigma^2\to 0} x_i^\star = 0$ *for all* i*: in the limit, the team always wins, and we set* $i^\ddagger = 0$*. If not, then define*

$$\textit{pivotal type} \equiv i^\ddagger \equiv \max\left\{ i \in \{1,\ldots,i^\circ\} \;:\; \sum_{j=i-1}^{i^\circ-1} |\mu_j|(\bar{U}_{j+1}-\bar{U}_j) \geq \sum_{j=i^\circ}^{n} \mu_j(\bar{U}_{j+1}-\bar{U}_j) \right\}. \tag{18}$$

The equilibrium thresholds satisfy $\lim_{\sigma^2\to 0} x_i^\star = x^\diamond$ *for* $i \geq i^\ddagger$*, and* $\lim_{\sigma^2\to 0}(x_i^\star - x_{i-1}^\star) > 0$ *for* $i \leq i^\ddagger$*.*

The proof of Lemma 4 goes further than the statement made here. Whereas the lemma says that the gaps between the thresholds of higher types vanish (specifically: $\lim_{\sigma^2\to 0}(x_{i+1}^\star - x_i^\star) = 0$ for $i \geq i^\ddagger$) the proof also calculates the rate at which those gaps vanish.[17] The inequality in eq. (18) is harder to satisfy when upward drift among good types is stronger and downward drift among bad types is weaker, which pushes the index of the pivotal type downward. This means that the properties of all recruit types can matter for the identification of the pivotal type that enables a winning team.

Corollary (to Lemma 4). *The* pivotal recruit type $i^\ddagger$ *is the lowest type whose participation triggers the participation of all higher types in the vanishing-noise limit. The identity of this pivotal type depends on the drift and outside-option parameters of all types, both good and bad. Consequently, even types that are never recruited in the vanishing-noise equilibrium can influence which recruit is pivotal.*

It is also true that the full range of recruit types matters for the determination of the critical threshold $x^\diamond$ that must be crossed for a team to be on a winning path. This is confirmed in our main proposition of this section, which characterizes that threshold.

[17] Specifically, the proof of Lemma 4 shows that if $i > i^\ddagger$ then

$$\lim_{\sigma^2\to 0} \frac{x_i^\star - x_{i-1}^\star}{\sigma^2} = \frac{\log(1+X_i)}{2\mu_{i-1}} \quad \text{where} \quad X_i = \frac{\mu_{i-1}(\bar{U}_i - \bar{U}_{i-1})}{\sum_{j=i}^{n} \mu_j(\bar{U}_{j+1}-\bar{U}_j)}. \tag{19}$$

Proposition 7 (Properties of Equilibrium as Noise Vanishes)**.** *Consider the unique equilibrium in a world where the team's status evolves via* $dx_t = \mu_i\, dt + \sigma\, dz_t$*.*

(i) If $i^\ddagger = 0$ *then* $x^\diamond = 0$*, and so* $\lim_{\sigma\to 0} x_i^\star = 0$ *for all* i*: the organization is always a winning team.*

(ii) Otherwise, if $i^\ddagger > 0$*, so that the team is not always winning, then*

$$x^\diamond = \bar{x}_{i^\ddagger-1} + \frac{|\mu_{i^\ddagger-1}|}{\rho}\log\left(\frac{(U-\bar{U}_{i^\ddagger-1})|\mu_{i^\ddagger-1}|}{\sum_{j=i^\ddagger}^{n}(\mu_j+|\mu_{i^\ddagger-1}|)(\bar{U}_{j+1}-\bar{U}_j)}\right). \tag{20}$$

(iii) If $i^\ddagger > 0$ *then the limiting threshold* $x^\diamond$ *is strictly decreasing in the drift rates of all types: it is decreasing in* μ_i *for all good types* $i \geq i^\circ$*, and increasing in* $|\mu_i|$ *for all bad types* $i < i^\circ$*.*

An immediate implication of claim (iii) is that the limiting equilibrium threshold responds monotonically to the drift parameters of all types. Eliminating a recruit type i is equivalent to lowering its drift from μ_i to μ_{i-1}, which strictly increases the limiting threshold $x^\diamond$. Conversely, introducing an additional recruit type with strictly higher drift strictly lowers $x^\diamond$. Thus, the presence of additional talent types—holding fixed the outside-option structure—shifts the equilibrium threshold downward.

Corollary (Stepping-Stone Effect)**.** *Adding an additional recruit type with drift strictly between two existing drift rates strictly lowers the limiting equilibrium threshold* $x^\diamond$*. Equivalently, eliminating such a type strictly raises* $x^\diamond$*. Additional "stepping stones" lower the threshold for a winning team.*

The stepping-stone effect operates through two distinct channels, depending on the location of the altered or newly introduced type relative to the pivotal recruit $i^\ddagger$. If the type lies strictly below $i^\ddagger$, the effect works through changes in the dynamics of losing teams. If instead the altered type lies at or above the pivotal recruit $i^\ddagger$, the effect operates through the collective dynamics of types whose thresholds collapse to the common limit $x^\diamond$. This set includes all good types and possibly some less-bad types, and its composition shapes how participation propagates once the pivotal threshold is crossed. We consider these two channels in turn.

When the altered type lies strictly below the pivotal recruit $i^\ddagger$, it is necessarily a type whose participation occurs in the losing region. Improving such a type (or introducing a new one) slows the rate of downward drift when the team is below the threshold, thereby increasing the continuation value $V(x)$ in that region. We refer to this mechanism as *downside smoothing*. By slowing the decline of a losing team, it cushions the downside risk faced by recruits near the threshold.

This channel mirrors the equilibrium-selection logic in the context of the simpler baseline model of Section 1. In the two-type benchmark, improving the drift of the losing state raises the career value below the threshold and alters the indifference condition that pins down equilibrium (see Figure 1). The same logic applies here. A higher continuation value in the losing region reduces the relative penalty from downward movements near the threshold and shifts the balance between upward and downward transitions. As a result, the limiting threshold $x^\diamond$ falls.

In the vanishing-noise limit, Lemma 4 implies that all types $i \geq i^\ddagger$ use the same participation threshold $x^\diamond$. This set includes all good types (as noted in the corollary to Proposition 5) and may include some lower types as well. In particular, once the pivotal type $i^\ddagger$ participates, all higher types immediately follow. Along the realized equilibrium path of the deterministic limit, the team effectively recruits only the elite type n, since participation of any type $i \geq i^\ddagger$ induces participation of all higher types.

Nevertheless, types in this collapsing set can still influence the limiting threshold. Improving or introducing a type $i \geq i^\ddagger$ raises the aggregate upward drift term appearing in Proposition 7. Even if the identity of $i^\ddagger$ remains unchanged, the summation terms in the expression for $x^\diamond$ are altered, lowering the equilibrium threshold. We refer to this mechanism as *upward triggering*: the presence of an additional type increases the strength of upward propagation once the pivotal threshold is reached, thereby reducing the status level required for participation.

Importantly, this effect does not operate through changes in the winning-region continuation value. Under the simplifying assumption $U = W$, the career value in the winning region of the deterministic limit is simply U, independent of the drift parameters of types in the collapsing set. Although status continues to rise at rate μ_n once the pivotal threshold is crossed, this does not affect the value function at the threshold itself. The upward-triggering channel therefore works entirely through the determination of $x^\diamond$, not through changes in the realized value of a winning team.

Corollary (Underwater Stepping Stones)**.** *Suppose $i \geq i^\ddagger$. Increasing μ_i or introducing an additional type with drift between μ_i and μ_{i+1} strictly lowers the limiting equilibrium threshold $x^\diamond$, even though such a type is never recruited along the equilibrium path of the deterministic limit.*

These types are "underwater" in the sense that they are never observed along the equilibrium path, yet their mere availability reshapes the threshold that determines whether the team ultimately succeeds.

Summary. Recruitment decisions remain strategically interdependent: each type anticipates how the participation of others affects team status and, in turn, the incentives of higher types to join. The presence of intermediate recruits reshapes these expectations and lowers equilibrium thresholds through downside smoothing and upward triggering. Heterogeneity in the potential talent pool can therefore be an ingredient of success.[18]

3. Expectations of Success as an Efficiency Wage

A team's prospects also depend on the effort choices of those who join. We now abstract from recruitment and consider equally talented team members who choose costly effort that affects the evolution of team status. Effort is driven by efficiency-wage concerns: it slows the arrival of career-ending shocks and is worthwhile when continued membership is sufficiently valuable. The feedback between effort and status generates multiple equilibria in the absence of noise. As before, introducing noise selects a unique threshold equilibrium.

[18]As an applied example, consider an academic department deciding how broadly to search for faculty. An adequate recruit recognizes that her participation may induce stronger candidates to join, thereby reinforcing upward drift; stronger candidates anticipate this reinforcement and become more willing to participate. Intermediate types thus facilitate the recruitment of elite talent. A related but distinct logic was developed by Mattozzi and Merlo (2008), who studied equilibrium recruitment of intermediate types; here, by contrast, intermediate types serve as stepping stones that enable the participation of elites.

A Model of Team Effort. The team's status now evolves according to its members' effort choices.

Players, Moves, and Payoffs. Team members (a unit mass, including replacements) choose at each time t whether to exert high ($i = 1$) or low ($i = 0$) effort. In equilibrium all members choose the same effort, and it suffices to describe dynamics when effort is uniform. Status $x_t \in [0, \bar{x}]$ evolves as

$$dx_t = \mu_i \, dt + \xi \sigma_i \, dz_t, \quad \text{with} \quad \mu_1 > 0 > \mu_0, \tag{21}$$

where we have added the scaling factor ξ for the noise term, to enable us to take a limit as $\xi \to 0$.

A team member's flow payoff depends upon the effort choice $i \in \{0, 1\}$. Using an obvious notation, $0 < u_1 < u_0$, so that effort is costly. A team member faces the Poisson arrival of a separation shock (and replacement with a substitute) with an exit payoff of zero. Higher effort slows the arrival: we write $\lambda_0 > \lambda_1 > 0$. As before, there are also winning and losing terminal payoffs W and L.

If a team member chooses effort i in a team that lasts forever then the present value of his career is $U_i \equiv u_i/(\lambda_i + \rho)$. We assume that $W \geq U_i \geq L$ for $i \in \{0, 1\}$ so that a team member actively wants the team to win and to avoid losing. We also assume that $W > (u_0 - u_1)/(\lambda_0 - \lambda_1) > L$. This implies that effort is exerted if and only if the team's prospects are sufficiently strong.

Solution Concept. We continue to seek Markov equilibria. Fixing the (Markovian) behavior of other team members, we write $V(x)$ for an individual career value when the team's status is x.

Effort Choice in an Evolving Team. Higher status moves a team closer to the winning line, and so $V(x)$ is increasing in x. The optimal effort decision depends upon the value of that career. Effort is costly, but slows the arrival of a forced departure; this second effect is stronger when career values are higher. It is optimal for a team member to exert effort if and only if

$$V(x) \geq \bar{U} \quad \text{where} \quad \bar{U} \equiv \frac{u_0 - u_1}{\lambda_0 - \lambda_1}. \tag{22}$$

This induces the same threshold structure as our talent-recruitment model: there is a threshold $x^\star$ such that $V(x) \gtrless \bar{U}$ if and only if $x \gtrless x^\star$. However, in a world with no noise (so that $\xi^2 = 0$) there are many possible solutions for $x^\star$, and so there are multiple equilibria.

With noise, a variant of Lemma 1 holds: the value function is smoothly and strictly increasing. A threshold strategy remains optimal, and the monotonicity arguments from Section 1 imply uniqueness.

Proposition 8 (Unique Equilibrium Effort Choice with Noisily Evolving Status)**.** *Consider a two-effort-level world in which team members' effort choices respond to career values.*

(i) There is a unique threshold $x^\star$ such that high effort is exerted if and only if status exceeds $x^\star$. This result extends to specifications in which drift rates or flow payoffs depend positively on status.

(ii) Allowing noise to vanish, the limiting threshold $x^\diamond = \lim_{\xi^2 \to 0} x^\star$ is increasing in σ_1^2/σ_0^2. As in the recruitment model, safety is valuable above the threshold, while risk facilitates recovery below it.

Claim (i) extends an earlier and simpler model (Dewan and Myatt, 2012) to a richer specification, while claim (ii) extends the "gambling for resurrection" insight to this efficiency-wage setting.[19]

[19] In Dewan and Myatt (2012) career values depend solely on the length of time in play (equivalent to eliminating the upper barrier $\bar{x}$ and $W = U$), actions are binary, and noise is homoscedastic.

An Effort Ladder to Team Success. We now extend this effort-choice model to multiple-effort levels indexed by $i \in \{0, 1, \dots, n\}$. We choose (as in Section 2) a homoscedastic specification,

$$dx_t = \mu_i \, dt + \sigma \, dz_t, \tag{23}$$

where μ_i is increasing in i, and $\mu_i > 0$ if and only if $i \geq i^\circ$ for some $i^\circ > 0$. This maps into our model of multiple recruit types. The flow payoff u_i from team membership is decreasing in i (so that effort is costly) and the hazard rate of separation λ_i is decreasing in effort. We also assume that

$$W \geq U_n > \bar{U}_n > \bar{U}_{n-1} > \cdots > \bar{U}_1 > 0 \quad \text{where} \quad \bar{U}_i \equiv \frac{u_{i-1} - u_i}{\lambda_{i-1} - \lambda_i}. \tag{24}$$

Here $\bar{U}_i$ is the critical efficiency wage (or career value) that induces effort i. Following our "stepping stones" model, team members care only about playing forever and there is no upper bound to the field of play. With these ingredients in hand, Propositions 6 and 7 generalize to this effort-choice context.

Proposition 9 (Unique Equilibrium Effort Choice with Noisily Evolving Status)**.** *Consider a multiple-effort-level world in which team members' effort choices respond to career values.*

(i) There is a unique equilibrium in which team members exert effort level of at least i when the team's status exceeds the threshold $x_i^\star$, where these thresholds satisfy $0 < x_1^\star < \cdots < x_n^\star$.

Now consider an environment in which there is no upper bound to the team's status, and so winning is the same as playing forever. (Formally: let $\bar{x} \to \infty$.)

(ii) Allowing noise to vanish, the thresholds for all efforts that satisfy $\mu_i > 0$ converge: there is some $x^\diamond$ such that $\lim_{\sigma^2 \to 0} x_i^\star = x^\diamond$ for all $i \geq i^\circ$. If status exceeds $x^\diamond$, effort jumps to the maximal level. However, the presence of intermediate effort levels strictly lowers this critical threshold.

Despite their availability, intermediate effort levels are again "underwater" stepping stones. Suppose $\mu_0 < 0 < \mu_1 < \cdots < \mu_n$, so that any positive effort generates upward drift. In the vanishing-noise limit equilibrium effort appears all-or-nothing: team members either exert no effort ($i = 0$) or the maximal effort ($i = n$). This might suggest that intermediate effort levels are irrelevant. However, they influence the critical threshold $x^\diamond$ that separates these regimes. The upward-triggering mechanism applies here: intermediate effort levels require a lower career value to be sustained, and the expectation that effort can rise progressively from lower to higher levels makes maximal effort credible at a lower status. Even though these intermediate effort levels are not observed along the equilibrium path in the limit, their presence lowers the threshold at which full effort is supplied.

Summary. This section extends the efficiency-wage framework of Dewan and Myatt (2012) to an environment with stochastic evolution and richer payoff structures. As in the recruitment model, noise selects a unique threshold equilibrium and generates asymmetric comparative-static effects.

Allowing for multiple effort levels reveals a further stepping-stone mechanism. Although equilibrium behavior appears all-or-nothing in the vanishing-noise limit, intermediate effort levels influence the critical threshold separating maximal and minimal effort. Even when unobserved along the equilibrium path, these intermediate levels lower the threshold at which full effort is supplied. Thus, permitting intermediate steps in effort can enhance long-run performance by reshaping expectations.

4. CONCLUSION

Organizational success can be governed by threshold dynamics. When participation and effort incentives depend on future prospects, strategic complementarities generate self-reinforcing paths in which optimistic expectations sustain success and pessimistic expectations induce decline. Introducing stochastic evolution selects a unique equilibrium and yields sharp comparative-static predictions for how payoffs, outside options, and volatility shape recruitment and effort provision.

Two implications stand out. Firstly, risk has asymmetric effects across states: volatility in successful regimes raises entry thresholds, whereas in declining regimes it lowers them. Uncertainty is costly when success is within reach but can be valuable when facing deterioration. Secondly, the structure of available talent (or the shape of a production technology in an effort-choice context) matters even when some types are not active or actions are not used in a deterministic equilibrium. Intermediate types and actions reshape continuation values and lower the thresholds required to attract elite recruits or motivate maximal effort, illustrating a stepping-stone mechanism through which even inactive types and actions influence long-run organizational outcomes. In summary: in dynamic coordination settings with heterogeneous types, intermediate states that are not observed in equilibrium nonetheless affect equilibrium selection by reshaping continuation values in the vanishing-noise limit.

More broadly, the framework (including, most notably, the "stepping stones" insights) can lend itself to many applications in which current participation depends on expectations about future viability. This includes sectoral choice and industrialization models with external economies (Matsuyama, 1991; Frankel and Pauzner, 2000), technology adoption with network effects (Katz and Shapiro, 1985, 1986; Farrell and Saloner, 1985; Guimaraes and Pereira, 2016), dynamic resource exploitation in which anticipated scarcity shapes current extraction (Kremer and Morcom, 2000; Tornell and Velasco, 1992; Rowat and Dutta, 2007), and team project dynamics (Georgiadis, 2015; Cvitanić and Georgiadis, 2016). In such settings, deterministic environments can admit multiple self-fulfilling paths, while stochastic evolution can select a unique threshold equilibrium. Our contribution highlights that the composition of intermediate states—talent types, adoption levels, or effort levels—drives equilibrium selection even when those states are not observed along a realized path.

Our talent-recruitment framework also extends naturally to settings with competing organizations. In ongoing companion work (Dewan and Myatt, 2026), we study a model in which talented recruits choose whom to serve rather than whether to participate and we derive a threshold characterization of a zone of contention in which both organizations recruit successfully. When relative status lies within this region, both teams are able to operate successfully; outside it, one organization attracts all talent. These results reinforce the central insight of this paper: expectations and the composition of available talent jointly determine organizational success and competitive outcomes.

Appendix A that follows contains proofs of most results reported in the main text. An optional on-line supplement contains two further appendices: Appendix B contains additional discussion of some literature beyond the brief commentary in the main text, while Appendix C reports the proof of Lemma 2 as well as the proof of several claims that are used in the proofs of Lemmas 3 and 4.

Appendix A. Omitted Proofs

Proof of Proposition 1. Equation (2) follows from the discussion in the text. By inspection, $V(x)$ is strictly increasing. Also as noted in the text, $x^\star$ generates an equilibrium if $V(x) \lesseqgtr \bar{U} \Leftrightarrow x \lesseqgtr x^\star$. This holds if $\lim_{x \uparrow x^\star} V(x) \leq \bar{U} \leq \lim_{x \downarrow x^\star} V(x)$, which is equivalent to eq. (3). These inequalities are

$$e^{-\rho(\bar{x}-x^\star)/|\mu_1|}(W-U) \geq \bar{U} - U \geq -e^{-\rho x^\star/|\mu_0|}(U-L). \tag{25}$$

These inequalities can be re-arranged to yield $x^\dagger \leq x^\star \leq x^\ddagger$ where

	if $\bar{U} < U$	if $\bar{U} > U$
$x^\dagger =$	0	$\max\left\{\bar{x} - \frac{\lvert\mu_1\rvert}{\rho}\log\left(\frac{W-U}{\bar{U}-U}\right), 0\right\}$
$x^\ddagger =$	$\min\left\{\frac{\lvert\mu_0\rvert}{\rho}\log\left(\frac{U-L}{U-\bar{U}}\right), \bar{x}\right\}$	$\bar{x}$

By inspection, $x^\dagger = 0$ and $x^\ddagger = \bar{x}$ if ρ is sufficiently small. The claims extend naturally to the richer specification in which the flow payoff from team membership and the drift rates of the team's status are both increasing functions of x, but the details are omitted here. □

Proof of Lemma 1. Strulovici and Szydlowski (2015) considered an optimal control problem with a one-dimensional, time-homogenous diffusion state. Here, there is no control problem but nevertheless their Assumptions 1–3 are satisfied. Their Theorem 1 shows that the value function is continuously differentiable and is "the unique solution with linear growth of the HJB equation" (Strulovici and Szydlowski, 2015, p. 1022). This implies the required properties of $V(x)$, and the HJB equation corresponds to eq. (65) below. The remaining claims are all straightforward. □

Proof of Proposition 2. Let us fix a threshold $x^\star$ used by others. A team member's payoff and the team's future prospects depend on the team's current status x. Here we measure status relative to the threshold. Defining $\Delta = x - x^\star$, this evolves according to

$$d\Delta_t = \mu_i(x^\star + \Delta_t)\,dt + \sigma_i\,dz_t \quad \text{where} \quad i = \begin{cases} 1 & \Delta_t > 0 \\ 0 & \Delta_t < 0 \end{cases} \tag{26}$$

Given the assumption that $\mu_i(x)$ is increasing in x, this means that an increase in $x^\star$ raises the upward drift rate of Δ, which moves a player toward higher payoffs. The flow payoff from team membership is $u(\Delta + x^\star)$ which is increasing in $x^\star$. In summary: for any Δ, a team member enjoys a higher flow payoff and a higher drift rate upwards if $x^\star$ (the threshold use by others) is higher. We already noted (in the text) that the winning line is closer and the losing line is further away (given Δ) when $x^\star$ is higher. We conclude that $V(\Delta + x^\star \mid x^\star)$ is increasing in $x^\star$. A special case, of course, is when $\Delta = 0$, which corresponds to a recruit observing current status that is equal to the threshold. We conclude that $\bar{V}(x)$ as defined in the text is increasing in x. By inspection, it is continuous, and the endpoint properties (at 0 and $\bar{x}$) stated in the text follow from Lemma 1.

The comparative-static claims follow similarly. For example, a shift upward in $u(x)$ raises career values and so increases $\bar{V}(x)$, which pushes down the intersection $x^\star$ of this function with $\bar{U}$. □

Proof of Lemma 2. This uses standard techniques: see Appendix C for a complete proof. □

Proof of Proposition 3. Differentiating the solution for $V_i(x)$ from eq. (74) (in Appendix C) yields

$$V_i'(x') = \frac{(V_i(x'') - U)(b_i^+ - b_i^-) + (V_i(x') - U)(b_i^- e^{-b_i^+(x''-x')} - b_i^+ e^{-b_i^-(x''-x')})}{e^{-b_i^-(x''-x')} - e^{-b_i^+(x''-x')}} \tag{27}$$

$$V_i'(x'') = \frac{(V_i(x'') - U)(b_i^+ e^{b_i^-(x''-x')} - b_i^- e^{b_i^+(x''-x')}) + (V_i(x') - U)(b_i^- - b_i^+)}{e^{b_i^+(x''-x')} - e^{b_i^-(x''-x')}}. \tag{28}$$

For $i = 0$, we compute the derivative at the top of $[0, x^\star]$ where $V_0(x'') = V(x^\star) = \bar{U}$ and $V_0(x') = V(0) = L$. Similarly, for $i = 1$ we do so at the bottom of $[x^\star, \bar{x}$ where $V_1(x') = V(x^\star) = \bar{U}$ and $V_1(x'') = V(\bar{x}) = W$. These substitutions into eqs. (27) and (28) produce

$$\lim_{x\downarrow x^\star} V'(x) = V_1'(x^\star) = \frac{(U - \bar{U})(b_1^+ e^{-b_1^-(\bar{x}-x^\star)} - b_1^- e^{-b_1^+(\bar{x}-x^\star)}) - (U - W)(b_1^+ - b_1^-)}{e^{-b_1^-(\bar{x}-x^\star)} - e^{-b_1^+(\bar{x}-x^\star)}} \quad \text{and} \tag{29}$$

$$\lim_{x\uparrow x^\star} V'(x) = V_0'(x^\star) = \frac{(U - L)(b_0^+ - b_0^-) - (U - \bar{U})(b_0^+ e^{b_0^- x^\star} - b_0^- e^{b_0^+ x^\star})}{e^{b_0^+ x^\star} - e^{b_0^- x^\star}}. \tag{30}$$

$V(x)$ is continuously differentiable at $x^\star$ (as required by Lemma 1) and so these expressions are equal. Equating them yields eq. (7) from the main text, which is repeated here

$$\frac{\bar{U}(b_0^+ e^{b_0^- x^\star} - b_0^- e^{b_0^+ x^\star}) - L(b_0^+ - b_0^-)}{e^{b_0^+ x^\star} - e^{b_0^- x^\star}} = \frac{W(b_1^+ - b_1^-) - \bar{U}(b_1^+ e^{-b_1^-(\bar{x}-x^\star)} - b_1^- e^{-b_1^+(\bar{x}-x^\star)})}{e^{-b_1^-(\bar{x}-x^\star)} - e^{-b_1^+(\bar{x}-x^\star)}}, \tag{31}$$

where (as in the main text) we set $U = 0$ (without loss of generality; payoffs are measured relative to the payoff from playing forever in the team) so that expressions remain compact. This is satisfied by the unique threshold $x^\star$. (For completeness, in Appendix C we check that this solution is unique.)

Our next step is evaluate this equilibrium condition as noise vanishes from the system. Incorporating ξ^2 into the various $b_i^\pm$ coefficients, taking the limit as $\xi^2 \to 0$ yields

$$\lim_{\xi\to 0} \xi^2 b_1^+ = \frac{2\mu_1}{\sigma_1^2}, \quad \lim_{\xi\to 0} b_1^- = -\frac{\rho}{\mu_1}, \quad \lim_{\xi\to 0} b_0^+ = -\frac{\rho}{\mu_0}, \quad \text{and} \quad \lim_{\xi\to 0} \xi^2 b_0^- = \frac{2\mu_0}{\sigma_0^2}. \tag{32}$$

The properties of b_1^+ and b_0^- are straightforward, while the claims regarding b_1^- and b_0^+ are obtained via l'Hôpital's rule. Noting that $b_0^- \to -\infty$ and so $e^{b_0^- x^\star} \to 0$ as $\xi^2 \to 0$,

$$\frac{\xi^2 \times \text{LHS of (31)}}{2} = \frac{1}{2} \times \frac{\bar{U}(\xi^2 b_0^+ e^{b_0^- x^\star} - \xi^2 b_0^- e^{b_0^+ x^\star}) - L(\xi^2 b_0^+ - \xi^2 b_0^-)}{e^{b_0^+ x^\star} - e^{b_0^- x^\star}} \tag{33}$$

$$\to \frac{\bar{U}|\mu_0|/\sigma_0^2 e^{\rho x^\star/|\mu_0|} - L|\mu_0|/\sigma_0^2}{e^{\rho x^\star/|\mu_0|}} = |\mu_0| \times \frac{\bar{U} - L e^{-\rho x^\star/|\mu_0|}}{\sigma_0^2} \tag{34}$$

as $\xi^2 \to 0$, where we used the properties of the $b_i^\pm$ coefficients derived above. This, for the normalization $U = 0$, is the left-hand side of eq. (9). The right-hand side of eq. (9) is obtained similarly.

The left-hand side is (by inspection) decreasing in $x^\diamond$, while the right-hand side is increasing in $x^\diamond$. Hence, the two sides cross at most once. Anything which raises the left-hand side or lowers the right-hand side pushes the intersection $x^\diamond$ to the right. The comparative-static claims follow. □

Proof of Proposition 4. Setting $W = U$ (winning is equivalent to playing forever) eq. (9) becomes

$$\frac{|\mu_0|((U - L)e^{-\rho x^\diamond/|\mu_0|} - (U - \bar{U}))}{\sigma_0^2} = \frac{\mu_1(U - \bar{U})}{\sigma_1^2}, \tag{35}$$

which solves to give the first explicit solution. $\bar{U} = U = 0$ yields the second explicit solution. □

Proof of Proposition 5. The claims made both in the proposition and in its corollary follow from the discussion in the main text, using an extension of the methods that underpin Proposition 1. □

Proof of Proposition 6. Recall that $X = \{\mathbf{x} \in [0, \bar{x}]^n : x_1^\star \leq \cdots \leq x_n^\star\}$ is the set of feasible threshold vectors, and $V(x \,|\, \mathbf{x})$ is the career value when the status is x and others use the thresholds in $\mathbf{x}$. From Lemma 1, $V(x \,|\, \mathbf{x})$ is continuous and strictly increasing in x, satisfying $V(0 \,|\, \mathbf{x}) = L$ and $V(\bar{x} \,|\, \mathbf{x}) = W$. We write $\mathrm{BR}(\mathbf{x}) \in X$ for the unique vector of thresholds that satisfy $V(\mathrm{BR}_i(\mathbf{x}) \,|\, \mathbf{x}) = \bar{U}_i$. Given that $V(x \,|\, \mathbf{x})$ is decreasing in $\mathbf{x}$, we note that if other recruits use thresholds weakly below those in $\mathbf{x}$ then the threshold $\mathrm{BR}_i(\mathbf{x}) \,|\, \mathbf{x})$ is strictly better for recruit i then any strictly higher threshold.

We now follow the procedure described in the main text. We set $\mathbf{x}^{(0)}$ to be the most pessimistic strategy profile in which all types use the threshold $\bar{x}$ (or higher) which means (in essence) that they never join the talent pool. We define iteratively $\mathbf{x}^{(s)} = \mathrm{BR}(\mathbf{x}^{(s-1)})$. Doing so, we eliminate strictly dominated strategies. This sequence is strictly decreasing and converges to the limit $\mathbf{x}^\star \equiv \lim_{s\to\infty} \mathbf{x}^{(s)}$ which satisfies (from continuity) $\bar{U}_i = V(x_i^\star \,|\, \mathbf{x}^\star)$ for all i.

Next, we follow the technique used by Frankel and Pauzner (2000). We build another sequence of threshold profiles, beginning with $\mathbf{x}^{(0)}$ equal to $x^\star$ translated downwards by $x_n^\star$. For type i, all thresholds strictly below $x_i^{(0)}$ are strictly dominated (where we note that using a negative threshold is equivalent to using a zero threshold). We then define iteratively $\mathbf{x}^{(0)}$ to satisfy

$$x_i^{(s)} = x_i^{(s-1)} + \min_{j \in \{1,\ldots,n\}} \left[\mathrm{BR}_j(\mathbf{x}^{(s-1)}) - x_j^{(s-1)} \right]. \tag{36}$$

Note that $\mathbf{x}^{(s)}$ is a translation upward from $\mathbf{x}^{(s-1)}$, and is a translation downward from $\mathbf{x}^\star$. For type i, all thresholds strictly below $x_i^{(s)}$ are strictly dominated by $x_i^{(s)}$ given that others use thresholds at or above those in $\mathbf{x}^{(s-1)}$. The sequence $\{\mathbf{x}^{(s)}\}$ iteratively eliminates strictly dominated strategies. It is a strictly increasing sequence, and so converges to some limit $\mathbf{x}^\bullet = \lim_{s\to\infty} \mathbf{x}^{(s)}$.

One possibility is that $\mathbf{x}^\bullet = \mathbf{x}^\star$, in which case $\mathbf{x}^\star$ is the unique equilibrium.

The other possibility is $\mathbf{x}^\bullet < \mathbf{x}^\star$. By construction, there is some j for whom $\mathrm{BR}_j(\mathbf{x}^\bullet) - x_j^\bullet$, and so for this type $\bar{U}_j = V(x_j^\bullet, \mathbf{x}^\bullet)$. But of course, $x^\star$ is an equilibrium and so $\bar{U}_j = V(x_j^\star, \mathbf{x}^\star)$, which implies that $V(x_j^\star, \mathbf{x}^\star) = V(x_j^\bullet, \mathbf{x}^\bullet)$. However, this yields a contradiction. To see why, note that a recruit type j observing status equal to the player's associated threshold expects the same relative evolution of status in both cases; this is because $\mathbf{x}^\bullet$ is a translation of $\mathbf{x}^\star$. However, at the threshold $x_j^\star$ the player is strictly further from the losing line and (under a more general specification in which flow payoffs increase with status) is closer to higher payoffs. This means that this player has a higher career value in a team with status $x_j^\star$ (given that others use the thresholds $\mathbf{x}^\star$) than in a team with status $x_j^\bullet$ (given that others use the thresholds $\mathbf{x}^\bullet$): $V(x_j^\star, \mathbf{x}^\star) > V(x_j^\bullet, \mathbf{x}^\bullet)$. This yields the desired contradiction. □

Proof of Lemma 3. The career value function consists of $n+1$ segments indexed by $i \in \{0, 1, \ldots, n\}$ where in the ith segment the drift is μ_i. Recall that we have (for simplicity of construction below; we can work without this assumption) removed the upper bound to the field of play so that $\bar{x} \to \infty$. For the nth segment $(x_n^\star, \infty)$ the value function satisfies $V_n(x) \to U$ as $x \to \infty$, and has solution

$$V_n(x) = U - (U - \bar{U}_n) e^{-b_n^+ (x - x_n^\star)}, \tag{37}$$

where this is obtained by taking $x'' \to \infty$ and $V_n(x'') \to U$ in eq. (74). Differentiation yields

$$V_n'(x_n^\star) = b_n^+(U - \bar{U}_n). \tag{38}$$

The ith segment (for $i < n$) is $(x_i^\star, x_{i+1}^\star)$ and satisfies $V_i(x_i^\star) = \bar{U}_i$ and $V_i(x_{i+1}^\star) = \bar{U}_{i+1}$ where we note that we have defined $\bar{U}_0 = L$. The solutions for the derivatives at the top and bottom of each segment, reported as eqs. (27) and (28) in the proof of Proposition 3, can be written as $V_i'(x_i^\star) = D_{i(-)}(y_{i+1}^\star)$ and $V_i'(x_{i+1}^\star) = D_{i(+)}(y_{i+1}^\star)$ where $y_{i+1}^\star \equiv x_{i+1}^\star - x_i^\star$ and where we define two auxiliary functions

$$D_{i(-)}(y) \equiv \frac{(U - \bar{U}_i)(b_i^+ e^{-b_i^- y} - b_i^- e^{-b_i^+ y}) - (U - \bar{U}_{i+1})(b_i^+ - b_i^-)}{e^{-b_i^- y} - e^{-b_i^+ y}} \tag{39}$$

$$\text{and} \quad D_{i(+)}(y) \equiv \frac{(U - \bar{U}_i)(b_i^+ - b_i^-) - (U - \bar{U}_{i+1})(b_i^+ e^{b_i^- y} - b_i^- e^{b_i^+ y})}{e^{b_i^+ y} - e^{b_i^- y}}. \tag{40}$$

Lemma 1 applies, and so the career value function is smooth. This means that the left-hand and right-hand derivatives must be equal when evaluated at each threshold. That is, for each $i \in \{1, \ldots, n\}$, $V_{i-1}'(x_i^*) = V_i'(x_i^*)$. Equivalently, and using the notation introduced above,

$$D_{(n-1)(+)}(y_n^\star) = b_n^+(U - \bar{U}_n) \quad \text{and} \tag{41}$$

$$D_{(i-1)(+)}(y_i^\star) = D_{i(-)}(y_{i+1}^\star) \quad \forall i \in \{1, \ldots, n-1\}. \tag{42}$$

We record some of the properties of these auxiliary functions here. These are proved in Appendix C.

Claims (Properties of the Auxiliary Functions)**.** *$D_{i(-)}(y)$ and $D_{i(+)}(y)$ have the following properties.*

(i) $D_{i(-)}(y)$ is strictly positive, $\lim_{y\downarrow 0} D_{i(-)}(y) = \infty$, and $\lim_{y\uparrow\infty} D_{i(+)}(y) = b_i^+(U - \bar{U}_i)$.

(ii) There is a $\bar{y}_i > 0$ at which $D_{i(+)}(y)$ crosses zero once from above to below: $D_{i(+)}(y) \gtrless 0$ if and only if $y \lessgtr \bar{y}_i$. $D_{i(+)}(y)$ is strictly decreasing for $y \leq \bar{y}_i$, and satisfies $\lim_{y\downarrow 0} D_{i(+)}(y) = \infty$. This implies the existence of a strictly and continuously decreasing inverse $D_{i(+)}^{-1}(\cdot) : [0, \infty) \mapsto (0, \bar{y}_i]$.

We are now able to solve iteratively for the gaps between the equilibrium thresholds. Noting the invertibility of $D_{i(+)}(\cdot)$, we solve eq. (41) for the gap between the top two thresholds:

$$y_n^\star = D_{(n-1)(+)}^{-1}\left(b_n^+(U - \bar{U}_n)\right), \tag{43}$$

and then, for $i < n$, we solve iteratively:

$$y_i^\star = D_{(i-1)(+)}^{-1}\left(D_{i(-)}(y_{i+1}^\star)\right), \tag{44}$$

where the properties for the auxiliary functions ensure that this procedure is well defined. Having solved for the gaps between the thresholds, we recover the thresholds themselves: $x_i^\star = \sum_{j=1}^{i} y_j^\star$.

With our equilibrium construction in hand, we are now able to proceed with the statements of the lemma. Note that, necessarily, $y_{i+1}^\star < \bar{y}_i$. For $i \geq i^\circ$ note that

$$\lim_{\sigma\to 0} \sigma^2 b_i^+ = 2\mu_i, \quad \text{and} \quad \lim_{\sigma\to 0} b_i^- = -\frac{\rho}{\mu_i},, \tag{45}$$

and so b_i^+ diverges to ∞ while b_i^- converges as $\sigma^2 \to 0$. Fixing $y > 0$, we will show that $D_{i(+)}(y)$ is negative for σ^2 sufficiently small. To do so, note that

$$D_{i(+)}(y) < 0 \quad \Leftrightarrow \quad \frac{U - \bar{U}_i}{U - \bar{U}_{i+1}} < \frac{b_i^+}{b_i^+ - b_i^-} \left(e^{b_i^- y} - \frac{b_i^- e^{b_i^+ y}}{b_i^+} \right). \tag{46}$$

Taking limits, we have

$$\lim_{\sigma^2 \to 0} \left[\frac{b_i^+}{b_i^+ - b_i^-} \left(e^{b_i^- y} - \frac{b_i^- e^{b_i^+ y}}{b_i^+} \right) \right] = e^{-\rho y/\mu_i} + \frac{\rho}{\mu_i} \lim_{b_i^+ \to \infty} \left[\frac{e^{b_i^+ y}}{b_i^+} \right] = \infty. \tag{47}$$

Hence eq. (46) holds for σ^2 small, which implies that $\bar{y}_i \to 0$. We conclude that $y_{i+1}^\star \to 0$ for all $i \geq i^\circ$. This implies that all thresholds $x_i^\star$ for $i \geq i^\circ$ must converge to the same limit. □

Lemma (Extended Statement of Lemma 4)**.** *If $i > i^\ddagger$, where $i^\ddagger$ is defined in the main lemma, then*

$$\lim_{\sigma^2 \to 0} \frac{x_i^\star - x_{i-1}^\star}{\sigma^2} = \frac{\log(1 + X_i)}{2\mu_{i-1}} \quad \textit{where} \quad X_i \equiv \frac{\mu_{i-1}(\bar{U}_i - \bar{U}_{i-1})}{\sum_{j=i}^n \mu_j (\bar{U}_{j+1} - \bar{U}_j)}. \tag{48}$$

Proof of (Extended) Lemma 4. We use further properties of the two auxiliary functions defined earlier.

Claims (Further Properties)**.** *$D_{i(-)}(y)$ and $D_{i(+)}(y)$ have the following properties.*

(i) Suppose that $\lim_{\sigma^2 \to 0} \sigma^2 D_{(i-1)(+)}(y_i^\star) = D_i \in (0, \infty)$. There are three cases: (a), (b), and (c). In case (a), where $i \geq i^\circ + 1$, or in case (b), where $i < i^\circ + 1$ and $D_i \geq 2|\mu_{i-1}|(\bar{U}_i - \bar{U}_{i-1})$, $y_i^\star$ satisfies

$$\lim_{\sigma^2 \to 0} \frac{y_i^\star}{\sigma^2} = \frac{1}{2\mu_{i-1}} \log \left(1 + \frac{2\mu_{i-1}(\bar{U}_i - \bar{U}_{i-1})}{D_i} \right). \tag{49}$$

However, in the third case (c), where, $i < i^\circ + 1$ and $D_i < 2|\mu_{i-1}|(\bar{U}_i - \bar{U}_{i-1})$, $y_i^\star$ satisfies

$$\lim_{\sigma^2 \to 0} y_i^\star = \frac{|\mu_{i-1}|}{\rho} \log \left(\frac{U - \bar{U}_{i-1}}{U - \bar{U}_i + D_i/(2|\mu_{i-1}|)} \right) > 0. \tag{50}$$

(ii) Next, consider $i < i^\circ$ and fix $y > 0$. Then:

$$\lim_{\sigma^2 \to 0} D_{i(-)}(y) = \frac{\rho(U - \bar{U}_i)}{|\mu_i|} \quad \Rightarrow \quad \lim_{\sigma^2 \to 0} \sigma^2 D_{i(-)}(y) = 0. \tag{51}$$

(iii) Also, and again for $i < i^\circ$ and fixing $y > 0$,

$$\lim_{\sigma^2 \to 0} \sigma^2 D_{i(+)}(y) = 2\mu_i \left((U - \bar{U}_{i+1}) - (U - \bar{U}_i) e^{-\rho y/|\mu_i|} \right). \tag{52}$$

(iv) Now fix $i \geq i^\diamond$ (a good recruit type) and again fix $y > 0$:

$$\lim_{\sigma^2 \to 0} \sigma^2 D_{i(+)}(y) = -\frac{\rho(U - \bar{U}_{i+1})}{\mu_i} \quad \Rightarrow \quad \lim_{\sigma^2 \to 0} \sigma^2 D_{i(+)}(y) = 0. \tag{53}$$

(v) Similarly,

$$\lim_{\sigma^2 \to 0} D_{i(-)}(y) = 2\mu_i \left((U - \bar{U}_i) - (U - \bar{U}_{i+1}) e^{-\rho y/\mu_i} \right). \tag{54}$$

(vi) Finally, we record another property of $D_{i(-)}(y_{i+1}^\star)$. If $\lim_{\sigma^2 \to 0}(y_{i+1}^\star/\sigma^2) = Y_{i+1} \in (0, \infty)$ then

$$\lim_{\sigma^2 \to 0} \sigma^2 D_{i(-)}(y_{i+1}^\star) = \frac{2\mu_i(\bar{U}_{i+1} - \bar{U}_i)}{1 - e^{-2\mu_i Y_{i+1}}}. \tag{55}$$

The proofs of these six claims are relegated to Appendix C.

The topmost gap $y_n^\star \equiv x_n^\star - x_{n-1}^\star$ eq. (43), or $\sigma^2 D_{(n-1)(+)}(y_n^\star) = \sigma^2 b_n^+(\bar{U}_{n+1} - \bar{U}_n)$. Given that $\mu_n > 0$, the coefficient b_n^+ has the property that $\sigma^2 b_n^+ \to 2\mu_n$ as $\sigma^2 \to 0$. We conclude that

$$\sigma^2 D_{(n-1)(+)}(y_n^\star) \to D_n \quad \text{where} \quad D_n \equiv 2\mu_n(\bar{U}_{n+1} - \bar{U}_n). \tag{56}$$

We now apply claim (i) stated above. Suppose that $n > i^\circ \geq i^\ddagger$. Case (a) of claim (i) applies. Hence:

$$\lim_{\sigma^2 \to 0} \frac{y_n^\star}{\sigma^2} = \frac{1}{2\mu_{n-1}} \log\left(1 + \frac{2\mu_{n-1}(\bar{U}_n - \bar{U}_{n-1})}{D_n}\right)$$
$$= \frac{1}{2\mu_{n-1}} \log\left(1 + \frac{\mu_{n-1}(\bar{U}_n - \bar{U}_{n-1})}{\mu_n(\bar{U}_{n+1} - \bar{U}_n)}\right) = \frac{\log(1 + X_n)}{2\mu_{n-1}}. \tag{57}$$

Suppose instead that $n = i^\circ > i^\ddagger$, where $i^\ddagger$ is defined in the lemma. Given $n = i^\circ > i^\ddagger$ we know that $|\mu_{n-1}|(\bar{U}_n - \bar{U}_{n-1}) < \mu_n(\bar{U}_{n+1} - \bar{U}_n)$ (to see this, use the definition of $i^\ddagger$ and the inequality in **??**) and so $2|\mu_{n-1}|(\bar{U}_n - \bar{U}_{n-1}) < D_n$. Case (b) of claim (i) applies and soeq. (57) holds.[20]

The final case to consider is when $n = i^\circ = i^\ddagger$. Part (c) of claim (i) now applies: $\lim_{\sigma^2 \to 0} y_n^\star > 0$.

We have established that the claims of the (extended version of the) lemma hold for $n = 1$.

Suppose (as an induction hypothesis) that the statements made in the lemma hold for all $\{i+1, \ldots, n\}$.

One possibility is that $\lim_{\sigma^2 \to 0} y_{i+1}^\star > 0$, which necessarily means that $i < i^\circ$. We can apply claim (ii) above, and conclude that $\sigma^2 D_{i(-)}(y_{i+1}^\star) \to 0$. This implies that $\sigma^2 D_{(i-1)(+)}(y_i^\star) \to 0$. Claim (iii) says that $\sigma^2 D_{(i-1)(+)}(y) \to 2\mu_i((U - \bar{U}_i) - (U - \bar{U}_{i-1})e^{-\rho y/|\mu_{i-1}|})$. From this we conclude that

$$U - \bar{U}_i = (U - \bar{U}_{i-1}) \lim_{\sigma^2 \to 0} e^{-\rho y_i^\star/|\mu_{i-1}|)} \quad \Leftrightarrow \quad \lim_{\sigma^2 \to 0} y_i^\star = \frac{|\mu_{i-1}|}{\rho} \log\left(\frac{U - \bar{U}_{i-1}}{U - \bar{U}_i}\right) > 0, \tag{58}$$

and so the claim of the lemma holds for i, just as it did for $i + 1$.

The other possibility is that $\lim_{\sigma^2 \to 0} y_{i+1}^\star = 0$. The induction hypothesis holds, and so

$$\lim_{\sigma^2 \to 0} \frac{y_{i+1}^\star}{\sigma^2} = Y_{i+1} \quad \text{where} \quad Y_{i+1} = \frac{\log(1 + X_{i+1})}{2\mu_i}. \tag{59}$$

Now we apply claim (vi) from above. Doing so, we find that

$$\lim_{\sigma^2 \to 0} \sigma^2 D_{i(-)}(y_{i+1}^\star) D_i \quad \text{where} \quad D_i = \frac{2\mu_i(\bar{U}_{i+1} - \bar{U}_i)}{1 - e^{-2\mu_i Y_{i+1}}} = \frac{2\mu_i(\bar{U}_{i+1} - \bar{U}_i)}{X_{i+1}/(1 + X_{i+1})}. \tag{60}$$

We then continue (as did for the inductive basis, when $i = n$) by applying claim (i) from above. For example, if $i \geq i^\circ + 1$ then case (a) of claim (i) applies and so

$$\lim_{\sigma^2 \to 0} \frac{y_i^\star}{\sigma^2} = \frac{1}{2\mu_{i-1}} \log\left(1 + \frac{2\mu_{i-1}(\bar{U}_i - \bar{U}_{i-1})}{D_i}\right) = \frac{1}{2\mu_{i-1}} \log\left(1 + \frac{\mu_{i-1}(\bar{U}_i - \bar{U}_{i-1})X_{i+1}}{\mu_i(\bar{U}_{i+1} - \bar{U}_i)(1 + X_{i+1})}\right)$$
$$= \frac{1}{2\mu_{i-1}} \log\left(1 + \frac{\mu_{i-1}(\bar{U}_i - \bar{U}_{i-1})}{\sum_{j=i}^n \mu_j(\bar{U}_{j+1} - \bar{U}_j)}\right) = \frac{\log(1 + X_i)}{2\mu_{i-1}}, \tag{61}$$

and so, once again, the lemma holds as stated for i, as it did for $i + 1$.

We have shown that the lemma holds for $i = n$. We have also shown that it holds for i, given that it holds for $\{i + 1, \ldots, n\}$. Hence, by the principle of induction, the lemma holds for all i. □

[20] The limit of $y_n^\star/\sigma^2$ is positive. Given $n = i^\circ$, $\mu_{n-1} < 0$, and so the denominator of the limit is negative. However, the numerator is also negative because $-1 < X_n < 0$ (as is readily checked) and the argument of the logarithm is below one.

Proof of Proposition 7. We established $y_i^\star \to 0$ for $i > i^\ddagger$ and that (via the extended Lemma 4) that

$$\frac{y_i^\star}{\sigma^2} \to \frac{\log(1+X_i)}{2\mu_{i-1}}. \tag{62}$$

In particular, this is true for $i^\ddagger + 1$. Applying claim (vi) from the proof of Lemma 4, we have

$$\sigma^2 D_{i^\ddagger(-)}(y^\star_{i^\ddagger+1}) \to \frac{2\mu_{i^\ddagger}(\bar{U}_{i^\ddagger+1} - \bar{U}_{i^\ddagger})}{1 - e^{-2\mu_{i^\ddagger} Y_{i^\ddagger+1}}} = \frac{2\mu_{i^\ddagger}(\bar{U}_{i^\ddagger+1} - \bar{U}_{i^\ddagger})(1 + X_{i^\ddagger+1})}{X_{i^\ddagger+1}} = 2\sum_{i=i^\ddagger}^{n} \mu_i(\bar{U}_{i+1} - \bar{U}_i). \tag{63}$$

Turning to $y^\star_{i^\ddagger}$, case (c) of claim (i) from the proof of Lemma 4 applies. Substituting in to the expression stated in that claim (via the term D_i) yields

$$y^\star_{i^\ddagger} \to \frac{|\mu_{i^\ddagger-1}|}{\rho} \log\left(\frac{(U - \bar{U}_{i^\ddagger-1})|\mu_{i^\ddagger-1}|}{\sum_{j=i^\ddagger}^{n}(\mu_j + |\mu_{i^\ddagger-1}|)(\bar{U}_{j+1} - \bar{U}_j)}\right), \tag{64}$$

and the remaining claims of the proposition follow immediately. □

Proof of Proposition 8. Claim (i) follows precisely the logic used to prove Proposition 2.

For claim (ii), Lemma 1 extends to this situation in which a team member endogenously chooses his effort level, and so the career value function smoothly and strictly increases from $V(0) = L$ to $V(\bar{x}) = W$, and satisfies $V(x^\star) = \bar{U}$ where $\bar{U}$ is defined in eq. (22). Above and below the threshold $x^\star$, the value function is readily solved explicitly. This solution, together with the limitation characterization, mimics earlier proofs and so the details are relegated to Appendix C. □

Proof of Proposition 9. The proof follows the steps used to prove Propositions 6 and 7. □

The bibliographic entries that follow include items referenced in our supplemental Appendix B.

REFERENCES

ACEMOGLU, D., G. EGOROV, AND K. SONIN (2010): "Political Selection and Persistence of Bad Governments," *Quarterly Journal of Economics*, 125(4), 1511–1575.

BANERJEE, A. V., AND R. PANDE (2007): "Parochial Politics: Ethnic Preferences and Politician Corruption," *CEPR Discussion Paper*, DP6381.

BEATH, A., F. CHRISTIA, G. EGOROV, AND R. ENIKOLOPOV (2014): "Electoral Rules and Political Selection: Theory and Evidence from a Field Experiment in Afghanistan," *Review of Economic Studies*, 83(3), 932–968.

BESLEY, T., AND S. COATE (1997): "An Economic Model of Representative Democracy," *Quarterly Journal of Economics*, 112(1), 85–114.

BROWNE, S. (1995): "Optimal Investment Policies for a Firm with a Random Risk Process: Exponential Utility and Minimizing the Probability of Ruin," *Mathematics of Operations Research*, 20(4), 937–958.

——— (1997): "Survival and Growth with a Liability: Optimal Portfolio Strategies in Continuous Time," *Mathematics of Operations Research*, 22(2), 468–493.

——— (1999): "Beating a Moving Target: Optimal Portfolio Strategies for Outperforming a Stochastic Benchmark," *Finance and Stochastics*, 3, 275–294.

BUISSERET, P., AND C. PRATO (2016): "Electoral Control and the Human Capital of Politicians," *Games and Economic Behavior*, 98, 34–55.

BURDZY, K., D. M. FRANKEL, AND A. PAUZNER (2001): "Fast Equilibrium Selection by Rational Players Living in a Changing World," *Econometrica*, 69(1), 163–189.

CARLSSON, H., AND E. VAN DAMME (1993): "Global Games and Equilibrium Selection," *Econometrica*, 61(5), 989–1018.

CASELLI, F., AND M. MORELLI (2004): "Bad Politicians," *Journal of Public Economics*, 88(3-4), 759–782.

CASPER, B. A., AND S. A. TYSON (2014): "Popular Protest and Elite Coordination in a Coup d'État," *Journal of Politics*, 76(2), 548–564.

CHENG, I. H., AND K. MILBRADT (2012): "The Hazards of Debt: Rollover Freezes, Incentives, and Bailouts," *Review of Financial Studies*, 25(4), 1070–1110.

CROUZET, N., A. GUPTA, AND F. MEZZANOTTI (2023): "Shocks and Technology Adoption: Evidence from Electronic Payment Systems," *Journal of Political Economy*, 131(11), 3003–3065.

CVITANIĆ, J., AND G. GEORGIADIS (2016): "Achieving Efficiency in Dynamic Contribution Games," *American Economic Journal: Microeconomics*, 8(4), 309–342.

DE MESQUITA, B. B., D. P. MYATT, A. SMITH, AND S. A. TYSON (2024): "The Punisher's Dilemma," *Journal of Politics*, 86(2), 395–411.

DEWAN, T., AND D. P. MYATT (2010): "The Declining Talent Pool of Government," *American Journal of Political Science*, 54(2), 267–286.

——— (2012): "Dynamic Government Performance: Honeymoons and Crises of Confidence," *American Political Science Review*, 106(1), 123–145.

——— (2026): "Competing for Talent: Relative Status, Zones of Contention, and the Survival of Rivals," *Working Paper*.

DIAMOND, D. W., AND P. H. DYBVIG (1983): "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 91(3), 401–419.

DIERMEIER, D., M. P. KEANE, AND A. MERLO (2005): "A Political Economy Model of Congressional Careers," *American Economic Review*, 95(1), 347–373.

DOWNS, G. W., AND D. M. ROCKE (1993): "Conflict, Agency, and Gambling for Resurrection: The Principal-Agent Problem Goes to War," *American Journal of Political Science*, 38(2), 362–380.

DUBINS, L. E., AND L. J. SAVAGE (1965): *How to Gamble If You Must: Inequalities for Stochastic Processes*. McGraw-Hill, New York.

DUTTA, P. K. (1994): "Bankruptcy and Expected Utility Maximization," *Journal of Economic Dynamics and Control*, 18(3–4), 539–560.

——— (1997): "Optimal Management of an R&D Budget," *Journal of Economic Dynamics and Control*, 21(2-3), 575–602.

ELLISON, G. (2000): "Basins of Attraction, Long-Run Stochastic Stability, and the Speed of Step-by-Step Evolution," *Review of Economic Studies*, 67(1), 17–45.

FARRELL, J., AND G. SALONER (1985): "Standardization, Compatibility, and Innovation," *RAND Journal of Economics*, 16(1), 70–83.

FOLKE, O., T. PERSSON, AND J. RICKNE (2016): "The Primary Effect: Preference Votes and Political Promotions," *American Political Science Review*, 110(3), 559–578.

FRANKEL, D. M., S. MORRIS, AND A. PAUZNER (2003): "Equilibrium Selection in Global Games with Strategic Complementarities," *Journal of Economic Theory*, 108, 1–44.

FRANKEL, D. M., AND A. PAUZNER (2000): "Resolving Indeterminacy in Dynamic Settings: The Role of Shocks," *Quarterly Journal of Economics*, 115(1), 285–304.

——— (2002): "Expectations and the Timing of Neighborhood Change," *Journal of Urban Economics*, 51(2), 295–314.

FRÉCHETTE, G. R., F. MANIQUET, AND M. MORELLI (2008): "Incumbents' Interests and Gender Quotas," *American Journal of Political Science*, 52(4), 891–909.

FRISELL, L. (2009): "A Theory of Self-Fulfilling Political Expectations," *Journal of Public Economics*, 93(5-6), 715–720.

GALASSO, V., AND T. NANNICINI (2011): "Competing on Good Politicians," *American Political Science Review*, 105(1), 79–99.

GEORGIADIS, G. (2015): "Projects and Team Dynamics," *Review of Economic Studies*, 82(1), 187–218.

GOLDSTEIN, I., AND A. PAUZNER (2005): "Demand-Deposit Contracts and the Probability of Bank Runs," *Journal of Finance*, 60(3), 1293–1327.

GUIMARAES, B., AND A. E. PEREIRA (2016): "QWERTY is Efficient," *Journal of Economic Theory*, 163, 819–825.

HARSTAD, B. (2012): "Buy Coal! A Case for Supply-Side Environmental Policy," *Journal of Political Economy*, 120(1), 77–115.

——— (2016): "The Dynamics of Climate Agreements," *Journal of Economic Theory*, 163, 386–423.

——— (2023): "Technology Adoption and Climate Policy," *Journal of Political Economy*, 131(5), 1270–1310.

HARSTAD, B., AND T. K. MIDEKSA (2017): "Conservation Contracts and Political Economy," *Review of Economic Studies*, 84(4), 1703–1734.

HE, Z., AND W. XIONG (2012): "Dynamic Debt Runs," *Review of Financial Studies*, 25(6), 1799–1843.

JÚLIO, P., AND J. TAVARES (2017): "The Good, the Bad, and the Different: Can Gender Quotas Raise the Quality of Politicians?," *Economica*, 84(335), 454–479.

KANDORI, M., G. J. MAILATH, AND R. ROB (1993): "Learning, Mutation, and Long Run Equilibria in Games," *Econometrica*, 61(1), 29–56.

KATZ, M. L., AND C. SHAPIRO (1985): "Network Externalities, Competition, and Compatibility," *American Economic Review*, 75(3), 424–440.

——— (1986): "Technology Adoption in the Presence of Network Externalities," *RAND Journal of Economics*, 17(4), 822–841.

KEANE, M. P., AND A. MERLO (2010): "Money, Political Ambition, and the Career Decisions of Politicians," *American Economic Journal: Microeconomics*, 2(3), 186–215.

KREMER, M., AND C. MORCOM (2000): "Elephants," *American Economic Review*, 90(1), 212–234.

LITTLE, A. T. (2012): "Elections, Fraud, and Election Monitoring in the Shadow of Revolution," *Quarterly Journal of Political Science*, 7(3), 249–283.

——— (2016): "Communication Technology and Protest," *Journal of Politics*, 76(1), 152–166.

——— (2017): "Coordination, Learning, and Coups," *Journal of Conflict Resolution*, 61(1), 204–234.

MAJUMDAR, S., AND S. W. MUKAND (2004): "Policy Gambles," *American Economic Review*, 94(4), 1207–1222.

MATSUYAMA, K. (1991): "Increasing Returns, Industrialization, and Indeterminacy of Equilibrium," *Quarterly Journal of Economics*, 106(2), 617–650.

MATTOZZI, A., AND A. MERLO (2008): "Political Careers or Career Politicians?," *Journal of Public Economics*, 92(3-4), 597–608.

——— (2015): "Mediocracy," *Journal of Public Economics*, 130, 32–44.

MATTOZZI, A., AND E. SNOWBERG (2018): "The Right Type of Legislator: A Theory of Taxation and Representation," *Journal of Public Economics*, 159, 54–65.

MORRIS, S., AND H. S. SHIN (1998): "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *American Economic Review*, 88(3), 587–597.

——— (2003): "Global Games: Theory and Applications," in *Advances in Economics and Econometrics, the Eighth World Congress*, ed. by M. Dewatripont, L. Hansen, and S. Turnovsky, pp. 56–114, Cambridge. Cambridge University Press.

——— (2004): "Coordination Risk and the Price of Debt," *European Economic Review*, 48(1), 133–153.

ROWAT, C., AND J. DUTTA (2007): "The Commons with Capital Markets," *Economic Theory*, 31(2), 225–254.

SMITH, G. J. (1967): "Optimal Strategy at Roulette," *Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete*, 8(2), 91–100.

STRÖMBERG, D. (2008): "How the Electoral College Influences Campaigns and Policy: The Probability of Being Florida," *American Economic Review*, 98(3), 769–807.

STRULOVICI, B., AND M. SZYDLOWSKI (2015): "On the Smoothness of Value Functions and the Existence of Optimal Strategies in Diffusion Models," *Journal of Economic Theory*, 159, 1016–1055.

SVOLIK, M. W. (2013): "Learning to Love Democracy: Electoral Accountability and the Success of Democracy," *American Journal of Political Science*, 57(3), 685–702.

TORNELL, A., AND A. VELASCO (1992): "The Tragedy of the Commons and Economic Growth: Why Does Capital Flow from Poor to Rich Countries?," *Journal of Political Economy*, 100(6), 1208–1231.

TYSON, S. A., AND A. SMITH (2018): "Dual-Layered Coordination and Political Instability: Repression, Co-optation, and the Role of Information," *Journal of Politics*, 80(1), 44–58.

YOUNG, H. P. (1993): "The Evolution of Conventions," *Econometrica*, 61(1), 57–84.

On-line Supplementary Material for "Playing for the Winning Team"

Torun Dewan · David P. Myatt · May 2026

Appendix B. Further Discussion of Related Literature (for On-Line Publication)

Here we offer additional remarks on related literature that go beyond commentary in the main text. (Note that the citations in this appendix refer to bibliographic entries reported in our main paper.)

The Selection Heuristic and Global Games. The equilibrium-selection heuristic of Section 1 can be related to logic used in the global games literature (Carlsson and van Damme, 1993; Frankel, Morris, and Pauzner, 2003). In a global game of the kind studied by Morris and Shin (1998, 2004) and others, there is an unknown state of the world that influences payoffs.[21] Beginning from a flat prior, players observe noisy signals of that fundamental before choosing their actions. In equilibrium, there is a threshold such that one action rather than its alternative is taken if and only if a player's signal exceeds the threshold. A property of beliefs in this setting is that each player thinks it equally likely that others' signals are above or below his own. This means that if a player's signal is equal to the threshold then he finds it equally likely that others will take the action or its alternative. Morris and Shin (2003) referred to such beliefs as Laplacian. Thus, an equilibrium threshold has the property that, given a signal equal to that threshold, a player with Laplacian beliefs about others (in essence: they expect moves into good and bad regimes to be equally likely) should be indifferent between his alternatives. This is closely connected to the heuristic logic that led us to eq. (5).

Similar connections between global games and a literature which studies dynamic games with frictions were made by Morris and Shin (2003) and by Frankel, Morris, and Pauzner (2003). From the latter literature, Frankel and Pauzner (2000) and Burdzy, Frankel, and Pauzner (2001) studied players who switch between alternatives, where the payoffs are influenced by a Brownian motion.

Frankel and Pauzner (2000) used a simplified version of the Matsuyama (1991) sector-choice model in which agents have occasional opportunities to switch between agricultural and manufacturing sectors. The former sector yields a constant flow payoff; an outside option in our setting. The latter sector exhibits external economies of scale: a switch to manufacturing is more valuable if others do so, either now or in the future. This generates multiple rational-expectations equilibria. To this, Frankel and Pauzner (2000) added noise (manufacturing productivity evolves according to a Brownian motion) and pinned down a unique equilibrium. They observed that in any equilibrium the relative dynamics for an agent are the same beginning from a point of indifference. This is the technique that generates our own uniqueness result. Naturally, there are many differences between our paper and theirs.[22]

[21] Morris and Shin (1998, 2004) applied global-games techniques to currency crises and debt pricing, while Goldstein and Pauzner (2005) found a unique prediction in a bank-run model based upon that of Diamond and Dybvig (1983). In political science there have been applications to protest, revolutions, and repression (for example, Casper and Tyson, 2014; Tyson and Smith, 2018; de Mesquita, Myatt, Smith, and Tyson, 2024; Little, 2012, 2016, 2017) and other phenomena, but there has not (to our knowledge) been extensive political-scientific use of the "noisy evolution" methods used here.

[22] For example, their manufacturing productivity parameter evolves separately to the agents' decisions. In our specification, agents' decisions and the evolving noise jointly determine the evolution of the team, with drift and volatility terms that depend upon recruits' decisions. The main focus of Frankel and Pauzner (2000) was to establish uniqueness. Here, however, we say much more about the determinants of the equilibrium threshold that separate winning and losing teams.

Non-Pecuniary Incentives in Organizations. A motivation for studying the "winning team" phenomena is that organizations in politics, sport, academia, and elsewhere might well rely on non-pecuniary or non-contractual incentives. In a political context, consider a government seeking to fill ministerial positions. A good recruit (or better politician) with enhanced human capital might enjoy a better outside option, perhaps in the private sector.[23] To map into our other payoff parameters, we interpret L to be the terminal payoff when a government loses office. If a minister cares only about his longevity then $W = U$ might be appropriate. If the organization is an opposition party, then we might set $U = L$ and W would be the gain when the opposition replaces the incumbent administration.

The multiplicity of equilibria (in the absence of noise) relates our work to that of Caselli and Morelli (2004). They explored how negative externalities (good politicians suffer from association with bad ones) and path dependence (office rewards depend on the actions of past office-holders) can generate multiple equilibria. They emphasized a coordination failure: an equilibrium in which the talent pool consists only of bad politicians. Here, the addition of noise to a government's evolving prospects pins down a unique equilibrium and so enables comparative-static results that provide new insights.

Others have also studied situations in which talent is scarce. In the model of Galasso and Nannicini (2011) a party allocates a limited pool of talented candidates to a larger set of districts. The resolution is to place the best politicians in the most competitive (marginal) districts.[24] Our own work (Dewan and Myatt, 2010) has explored the relationship between the depth of a government's talent pool and the incentives chosen by a leader.[25] Whereas these earlier papers assumed an exogenous talent pool, here the focus is on strategic complementarities and the endogenous depth of that talent pool.

Our theme is also shared by extensions of Caselli and Morelli (2004) which emphasize bad equilibria. For example, Fréchette, Maniquet, and Morelli (2008) and Júlio and Tavares (2017) analyzed the effect of quota restrictions on quality. Relatedly, Svolik (2013) argued that imperfect screening may result in the selection of crooked candidates who seize a one-time chance to exploit the public. A "trap of pessimistic expectations" can open in which voters perceive all politicians as crooks, and so politicians of every type exploit their offices. Prophesies of this kind were also analyzed by Frisell (2009) who showed that the public trust or distrust in politicians' behavior may be self-fulfilling.

Our application to political selection and performance relates to other research with this theme. Buisseret and Prato (2016) argued that better elected candidates need not generate higher voter welfare when they face competing claims for their time from party or faction. Acemoglu, Egorov, and Sonin (2010) also related an executive's performance to the quality of its composition. In their model incompetent incumbents can veto (desirable to voters) transitions that foreclose their participation in future governments. As a consequence a government that consists of the least competent ministers may persist. We show that this outcome can arise even in the absence of such restrictive institutions.[26]

[23] We note that skills attained in politics are transferrable into better outside options such as private-sector earning potential, and vice versa (Diermeier, Keane, and Merlo, 2005; Keane and Merlo, 2010).

[24] They documented empirical support, and revealed that talent is related to improved legislative performance.

[25] We found that performance, the depth of the talent pool, and (endogenous) incentives all weaken over time.

[26] A related literature has noted that elections may fail to deliver good politicians owing to the intermediary effects of parties (Mattozzi and Merlo, 2015). Additionally, voters may be unable to coordinate on the best candidate (Besley and Coate, 1997), may emphasize other dimensions over competence (Banerjee and Pande, 2007), or may face trade-offs between competence and ideology (Beath, Christia, Egorov, and Enikolopov, 2014; Mattozzi and Snowberg, 2018). Relatedly, Folke, Persson, and Rickne (2016) analyzed data connecting intra-party competition with quality.

Bold Play: Gambling for Resurrection. The result that it can pay to take risky gambles is a celebrated finding of Dubins and Savage (1965). They studied optimal strategies in casino games where a gambler survives only if his wealth crosses a target threshold. If the odds are against him then bold play—a maximum stake on a single outcome, so long as the winning outcome does not exceed the threshold—is optimal, whereas timid play—spreading bets—-leads inevitably to his ruin. The "bold play" strategy was extended to more general roulette games by Smith (1967). Other extensions have explored how the adoption of risky strategies depends upon the initial level of resources. For example, Dutta (1994) studied a gambler's ruin problem in which an expected-utility-maximizing agent facing the possibility of bankruptcy must balance utility against survival.[27] Relatedly, Dutta (1997) evaluated when bold play is optimal in the use of a limited research-and-development budget. Other work has, for example, used the bold-play idea to examine portfolio strategies (Browne, 1995, 1997, 1999).

Our comparative-static result (Proposition 3) provides insights into recruitment strategies: a recruiter might trade lower expected talent for less risk if the team is winning, but favors greater risk if it is losing. Similar trade-offs have been studied elsewhere. Downs and Rocke (1993) analyzed risky foreign policy ventures: a leader "gambles on resurrection" by continuing with an adventurous war because "cessation would ... cause him to be removed from office." Policy gambles were also studied by Majumdar and Mukand (2004) who showed that an incumbent may make inefficient policy choices when a voter judges ability from policy outcomes. The mean-variance trade off in our model also relates to work by Strömberg (2008). He studied campaigning across safe and less safe seats.

Intermediate Types and Mediocracy. Our "stepping stones" are central to our paper: the existence of intermediate (perhaps mediocre) types can be beneficial. A related point was made by Mattozzi and Merlo (2008). They noted that "politicians are typically not the best a country has to offer. At the same time, however, it is also fair to say that they are not the worst either." In their two-party model, each party recruits from types that are heterogenous with respect to the marginal cost of effort. Recruits choose effort that generates rents (from total effort) for the party which in turn chooses as leader the individual exerting the greatest effort. The equilibrium recruitment strategy involves a "mediocracy" in which parties choose intermediate politicians even when more productive individuals are willing to serve. Here, by contrast, the addition of mediocre politicians to the mix has a knock-on effect in making higher quality candidates available—the former are mere stepping stones for the latter.

Self-Fulfilling Prophesies in Resource Depletion. In our concluding remarks we noted that coordination in resource depletion could be an application of our framework.[28] For example, Kremer and Morcom (2000) modeled the harvesting of storable open-access resources. There are multiple equilibria. If a resource is expected to survive then present prices (via the possibility of storage) will be low, exploitation will be dampened, and so the resource survives. However, if the resource is expected to disappear, then high prices in both the future and present result in the high harvesting rates that are consistent with the depletion. Similarly, Rowat and Dutta (2007) analyzed commons exploitation in the presence of capital markets and found multiple equilibria.

[27] At each moment the agent's action influences the drift and volatility of his stochastically evolving wealth. As his wealth increases, the optimal variance increases (or rather the mean/variance ratio decreases).

[28] Expectations about anticipated future scarcity can shape current extraction. Relatedly, anticipated policy change can also feedback into current behavior (Harstad, 2012, 2016, 2023; Harstad and Mideksa, 2017).

APPENDIX C. SUPPLEMENTARY PROOFS (FOR ON-LINE PUBLICATION)

Proof of Lemma 2. The HJB equation (mentioned in the proof to Lemma 1) for a team member is

$$u - \rho V(x) + \mu_i V'(x) + \frac{\sigma_i^2 V''(x)}{2} = 0. \tag{65}$$

This can be solved straightforwardly to yield eq. (6). For completeness we sketch the derivation here. The value function $V(x)$ must satisfy

$$\rho V(x)\,dt = u\,dt + \mathrm{E}[dV(x)]. \tag{66}$$

The left-hand side is the flow rental value of a career within the team. The first term on the right-hand side is the direct flow benefit from that position. The second term on the right-hand side is the expected flow change in the value of the career.

The value function consists of two segments: above and below the threshold. We write $V_i(x)$ for the solution to the value function in each of those two segments $i \in \{0, 1\}$. For each segment of the career value function we can use Itô's Lemma:

$$\mathrm{E}[dV_i(x)] = \mu_i V_i'(x)\,dt + \frac{\sigma_i^2 V_i''(x)}{2}\,dt. \tag{67}$$

The first term on the right-hand side corresponds to the expected path of the team's status, which influences the career value of a team member via the derivative $V_i'(x)$. There is also a second-order effect owing to the presence of the random component. This is captured by the second term, which depends upon the curvature of the value function. Bringing together eqs. (66) and (67), we obtain

$$u - \rho V_i(x) + \mu_i V_i'(x) + \frac{\sigma_i^2 V_i''(x)}{2} = 0. \tag{68}$$

This constant-coefficient linear second-order differential equation has a general solution

$$V_i(x) = a_i^+ e^{-b_i^+ x} + a_i^- e^{-b_i^- x} + U \quad \text{where} \quad U \equiv \frac{u}{\rho} \quad \text{and} \quad b_i^\pm \equiv \frac{\mu_i \pm \sqrt{\mu_i^2 + 2\rho\sigma_i^2}}{\sigma_i^2}. \tag{69}$$

The coefficients a_i^+ and a_i^- are determined by boundary conditions. For example, suppose that $V_i(x)$ is known at x' and x'' where $x' < x''$. Solving linearly yields

$$a_i^- = \frac{(V_i(x'') - U)e^{b_i^+ x''} - (V_i(x') - U)e^{b_i^+ x'}}{e^{(b_i^+ - b_i^-)x''} - e^{(b_i^+ - b_i^-)x'}} \tag{70}$$

$$a_i^+ = \frac{(V_i(x') - U)e^{b_i^- x'} - (V_i(x'') - U)e^{b_i^- x''}}{e^{-(b_i^+ - b_i^-)x'} - e^{-(b_i^+ - b_i^-)x''}} \tag{71}$$

Substituting back on gives us (conditional on these boundary conditions) the full solution

$$V_i(x) - U = \frac{(V_i(x') - U)e^{b_i^- x'} - (V_i(x'') - U)e^{b_i^- x''}}{e^{-(b_i^+ - b_i^-)x'} - e^{-(b_i^+ - b_i^-)x''}} e^{-b_i^+ x} \tag{72}$$

$$+ \frac{(V_i(x'') - U)e^{b_i^+ x''} - (V_i(x') - U)e^{b_i^+ x'}}{e^{(b_i^+ - b_i^-)x''} - e^{(b_i^+ - b_i^-)x'}} e^{-b_i^- x}. \tag{73}$$

This can be readily re-arranged to yield

$$V_i(x) - U = \frac{(V_i(x'') - U)(e^{-b_i^-(x - x')} - e^{-b_i^+(x - x')})}{e^{-b_i^-(x'' - x')} - e^{-b_i^+(x'' - x')}} + \frac{(V_i(x') - U)(e^{b_i^+(x'' - x)} - e^{b_i^-(x'' - x)})}{e^{b_i^+(x'' - x')} - e^{b_i^-(x'' - x')}}. \tag{74}$$

Consider now the low segment of the value function, where $x < x^\star$ and so $i = 0$. This corresponds to the segment with endpoints $x' = 0$ and $x'' = x^\star$ and with associated career values $V_0(x') = V(0) = L$ and $V_0(x'') = V(x^\star) = \bar{U}$. Substituting these into eq. (74) yields

$$V_0(x) - U = \frac{(\bar{U} - U)(e^{-b_0^- x} - e^{-b_0^+ x})}{e^{-b_0^- x^\star} - e^{-b_0^+ x^\star}} + \frac{(L - U)(e^{b_0^+ (x^\star - x)} - e^{b_0^- (x^\star - x)})}{e^{b_0^+ x^\star} - e^{b_0^- x^\star}}. \quad (75)$$

This, of course, is the first segment (for $x < x^\star$) reported in the statement of the lemma. The solution for the second segment (for $x > x^\star$, and so $i = 1$) can be obtained similarly. □

Fuller Proof of Proposition 3. We noted that eq. (31) is satisfied by the unique equilibrium threshold $x^\star$. For completeness, here we check that this solution is indeed unique. To see this, we first differentiate the left-hand side of eq. (31) to obtain

$$\frac{\partial[\text{LHS of (31)}]}{\partial x^\star} = -\frac{\bar{U}(b_0^+ e^{b_0^- x^\star} - b_0^- e^{b_0^+ x^\star}) - L(b_0^+ - b_0^-)}{e^{b_0^+ x^\star} - e^{b_0^- x^\star}} \times \frac{b_0^+ e^{b_0^+ x^\star} - b_0^- e^{b_0^- x^\star}}{e^{b_0^+ x^\star} - e^{b_0^- x^\star}} + \frac{\bar{U} b_0^+ b_0^- (e^{b_0^- x^\star} - e^{b_0^+ x^\star})}{e^{b_0^+ x^\star} - e^{b_0^- x^\star}}. \quad (76)$$

We note that $b_0^+ > 0 > b_0^-$ and so $e^{b_0^+ x^\star} - e^{b_0^- x^\star} > 0$. Hence this is negative if and only if

$$\begin{aligned} &L(b_0^+ - b_0^-)(b_0^+ e^{b_0^+ x^\star} - b_0^- e^{b_0^- x^\star}) \\ &\quad < \bar{U}\left[(b_0^+ e^{b_0^- x^\star} - b_0^- e^{b_0^+ x^\star})(b_0^+ e^{b_0^+ x^\star} - b_0^- e^{b_0^- x^\star}) + b_0^+ b_0^- (e^{b_0^+ x^\star} - e^{b_0^- x^\star})^2\right] \\ &\quad = \bar{U}(b_0^+ - b_0^-)^2 e^{(b_0^+ + b_0^-) x^\star} \quad \Leftrightarrow \quad L(b_0^+ e^{b_0^+ x^\star} - b_0^- e^{b_0^- x^\star}) < \bar{U}(b_0^+ - b_0^-) e^{(b_0^+ + b_0^-) x^\star}. \end{aligned} \quad (77)$$

We note that $b_0^+ e^{b_0^+ x^\star} - b_0^- e^{b_0^- x^\star} > 0$ and $b_0^+ - b_0^- > 0$. Recall that we have set $U = 0$ which implies that $L < 0$. Hence, if $\bar{U} > 0$ (which means that $\bar{U} > U$; this says that a recruit prefers his outside option to playing forever in the team, and so is motivated to join the team only by the prospect of winning the terminal prize W) then this inequality is always satisfied.

We turn to the case $L < \bar{U} < 0$. Multiplying both sides of the inequality by -1 (and so reversing the inequality) and writing $-L$ and $-\bar{U}$ as absolute values $|L|$ and $|\bar{U}|$, this inequality becomes

$$|L|(b_0^+ e^{b_0^+ x^\star} - b_0^- e^{b_0^- x^\star}) > |\bar{U}|(b_0^+ - b_0^-) e^{(b_0^+ + b_0^-) x^\star}. \quad (78)$$

Noting that $|L| > |\bar{U}|$ for this case where $L < \bar{U} < 0$, we see that a sufficient condition for this inequality to hold is that $(b_0^+ e^{b_0^+ x^\star} - b_0^- e^{b_0^- x^\star}) > (b_0^+ - b_0^-) e^{(b_0^+ + b_0^-) x^\star}$. This inequality is equivalent to requiring that $B(x^\star) > 0$ where the function $B(\cdot)$ is defined as

$$B(y) \equiv b_i^+ (e^{-b_i^- y} - 1) - b_i^- (e^{-b_i^+ y} - 1). \quad (79)$$

$B(0) = 0$, $B'(y) = -b_i^- b_i^+ (e^{-b_i^- y} - e^{-b_i^+ y}) \geq 0$, and $B'(y) > 0$ for all $y > 0$. Hence $B(x^\star) > 0$. Summarizing: we have shown that the left-hand side of eq. (31) is decreasing in $x^\star$.

We now study the right-hand side of eq. (31).

$$\frac{\partial[\text{RHS of (31)}]}{\partial x^\star} = -\frac{\bar{U} b_1^+ b_1^- (e^{-b_1^- (\bar{x} - x^\star)} - e^{-b_1^+ (\bar{x} - x^\star)})}{e^{-b_1^- (\bar{x} - x^\star)} - e^{-b_1^+ (\bar{x} - x^\star)}}$$

$$+\frac{W(b_1^+ - b_1^-) - \bar{U}(b_1^+ e^{-b_1^-(\bar{x}-x^\star)} - b_1^- e^{-b_1^+(\bar{x}-x^\star)})}{e^{-b_1^-(\bar{x}-x^\star)} - e^{-b_1^+(\bar{x}-x^\star)}} \times \frac{b_1^+ e^{-b_1^+(\bar{x}-x^\star)} - b_1^- e^{-b_1^-(\bar{x}-x^\star)}}{e^{-b_1^-(\bar{x}-x^\star)} - e^{-b_1^+(\bar{x}-x^\star)}}. \quad (80)$$

Algebraic manipulations similar to those above show that this is positive if and only if

$$\bar{U} e^{-(b_1^+ + b_1^-)(\bar{x}-x^\star)}(b_1^+ - b_1^-) < W(b_1^+ e^{-b_1^+(\bar{x}-x^\star)} - b_1^- e^{-b_1^-(\bar{x}-x^\star)}). \quad (81)$$

The normalization $U = 0$ implies that $W > 0$, and so the right-hand side is positive. The left-hand side has the sign of $\bar{U}$. Hence, if $\bar{U} < 0$ (which corresponds to $\bar{U} < U$, which says that recruit prefers perpetual play in the team to his outside option) then the left-hand side is negative, and so the inequality holds. The other possibility is that $0 < \bar{U} < W$, so that both sides of the inequality are positive. For this case, a sufficient condition for this to hold is $b_1^+ e^{-b_1^+(\bar{x}-x^\star)} - b_1^- e^{-b_1^-(\bar{x}-x^\star)} > e^{-(b_1^+ + b_1^-)(\bar{x}-x^\star)}(b_1^+ - b_1^-)$, which holds for all $\bar{x} - x^\star > 0$ using the same procedure as before.

Summarizing: we have shown that the right-hand side of eq. (31) is increasing in $x^\star$, while the left-hand side is decreasing in $x^\star$. This means that the two sides cross at most once. We need only confirm that these expressions do cross for some $x^\star \in [0, \bar{x}]$. To do this, note that

$$\lim_{x^\star \downarrow 0}[\text{LHS of (31)}] = \infty > \frac{W(b_1^+ - b_1^-) - \bar{U}(b_1^+ e^{-b_1^- \bar{x}} - b_1^- e^{-b_1^+ \bar{x}})}{e^{-b_1^- \bar{x}} - e^{-b_1^+ \bar{x}}} = \lim_{x^\star \downarrow 0}[\text{RHS of (31)}], \quad (82)$$

and so the left-hand side exceeds the right-hand side. Similarly,

$$\lim_{x^\star \uparrow \bar{x}}[\text{LHS of (31)}] = \frac{\bar{U}(b_0^+ e^{b_0^- \bar{x}} - b_0^- e^{b_0^+ \bar{x}}) - L(b_0^+ - b_0^-)}{e^{b_0^+ \bar{x}} - e^{b_0^- \bar{x}}} < \infty = \lim_{x^\star \uparrow \bar{x}}[\text{RHS of (31)}], \quad (83)$$

and so the right-hand sides exceeds the left-hand side. We conclude that the two sides of the two sides must cross in the interval $(0, \bar{x})$. Thus, eq. (31) pins down the unique equilibrium threshold. □

Proof of Claims (i) and (ii) from the Proof of Lemma 3. For claim (i), the denominator of $D_{i(-)}(y)$ is strictly positive, and so the sign is determined by the numerator. That numerator is strictly increasing in y, and is equal to $(b_i^+ - b_i^-)(\bar{U}_{i+1} - \bar{U}_i) > 0$ at $y = 0$. We conclude that $D_{i(-)}(y)$ is positive everywhere. The denominator vanishes as $y \to 0$ and so $D_{i(-)}(y)$ diverges to ∞. Taking the limit as $y \to \infty$, terms involving $e^{-b_i^- y}$ dominate others, which generates the remaining statement.

For claim (ii), we first note that the denominator of $D_{i(+)}(y)$ is strictly positive for $y > 0$, and so the sign of $D_{i(+)}(y)$ is determined by its numerator. Evaluated at $y = 0$, that numerator is equal to $(b_i^+ - b_i^-)(\bar{U}_{i+1} - \bar{U}_i) > 0$, it is strictly decreasing in y, and diverges to $-\infty$ as $y \to \infty$. This all implies that $D_{i(+)}(y)$ is strictly positive if and only if $y \le \bar{y}_i$ where $\bar{y}_i$ satisfies

$$(U - \bar{U}_i)(b_i^+ - b_i^-) = (U - \bar{U}_{i+1})(b_i^+ e^{b_i^- \bar{y}_i} - b_i^- e^{b_i^+ \bar{y}_i}). \quad (84)$$

The denominator vanishes as $y \to 0$ and so $D_{i(+)}(y)$ diverges to ∞ as claimed. Within $(0, \bar{y}_i)$ the numerator is strictly decreasing and positive, while the denominator is strictly increasing. This yields the monotonicity of $D_{i(+)}(y)$. The existence and properties of the inverse follow immediately. □

Proof of Claims (i)–(vi) from the Proof of Lemma 4. For (i), applying the definition of $D_{(i-1)(+)}(y)$:

$$\sigma^2 D_{(i-1)(+)}(y_i^\star) = \frac{(U - \bar{U}_{i-1})(\sigma^2 b_{i-1}^+ - \sigma^2 b_{i-1}^-) - (U - \bar{U}_i)(\sigma^2 b_{i-1}^+ e^{b_{i-1}^- y_i^\star} - \sigma^2 b_{i-1}^- e^{b_{i-1}^+ y_i^\star})}{e^{b_{i-1}^+ y_i^\star} - e^{b_{i-1}^- y_i^\star}}. \quad (85)$$

If $i \geq i^\circ + 1$ then $i-1 \geq i^\circ$ and so $\mu_{i-1} > 0$. This implies $\sigma^2 b^+_{i-1} \to 2\mu_{i-1}$, and $\sigma^2 b^-_{i-1} \to 0$. Hence:

$$D_i = 2\mu_{i-1} \lim_{\sigma^2 \to 0} \left[\frac{(U - \bar{U}_{i-1}) - (U - \bar{U}_i) e^{b^-_{i-1} y^\star_i}}{e^{b^+_{i-1} y^\star_i} - e^{b^-_{i-1} y^\star_i}} \right] + (U - \bar{U}_i) \lim_{\sigma^2 \to 0} \left[\frac{\sigma^2 b^-_{i-1} e^{b^+_{i-1} y^\star_i}}{e^{b^+_{i-1} y^\star_i} - e^{b^-_{i-1} y^\star_i}} \right]. \quad (86)$$

We know that $y^\star_i \to 0$ (this is part of the statement of Lemma 3) and b^-_{i-1} converges as $\sigma^2 \to 0$. Hence $b^-_{i-1} y^\star_i \to 0$, which in turn implies that $e^{b^-_{i-1} y^\star_i} \to 1$. So:

$$D_i = 2\mu_{i-1} \lim_{\sigma^2 \to 0} \left[\frac{\bar{U}_i - \bar{U}_{i-1}}{e^{b^+_{i-1} y^\star_i} - 1} \right] + (U - \bar{U}_i) \lim_{\sigma^2 \to 0} \left[\frac{\sigma^2 b^-_{i-1} e^{b^+_{i-1} y^\star_i}}{e^{b^+_{i-1} y^\star_i} - 1} \right]. \quad (87)$$

This equality is inconsistent with either $b^+_{i-1} y^\star_i \to 0$ (the right-hand side diverges) or $b^+_{i-1} y^\star_i \to \infty$ (the right-hand side converges to zero). We conclude that $\lim_{\sigma^2 \to \infty} b^+_{i-1} y^\star_i \in (0, \infty)$, which in turn (noting that $\sigma^2 b^-_{i-1} \to 0$) implies that the second term above converges to zero. Hence

$$D_i = 2\mu_{i-1} \lim_{\sigma^2 \to 0} \left[\frac{\bar{U}_i - \bar{U}_{i-1}}{e^{b^+_{i-1} y^\star_i} - 1} \right] \quad \Rightarrow \quad \lim_{\sigma^2 \to 0} b^+_{i-1} y^\star_i = \log\left(1 + \frac{2\mu_{i-1}(\bar{U}_i - \bar{U}_{i-1})}{D_i}\right). \quad (88)$$

Of course, $b^+_{i-1} y^\star_i = (\sigma^2 b^+_{i-1})(y^\star_i / \sigma^2)$, and $\sigma^2 b^+_{i-1} \to 2\mu_{i-1}$. This yields case (a) of claim (i).

Remaining with claim (i), consider now the case where $i < i^\circ + 1$ so that $i - 1 < i^\dagger$. Recruit $i-1$ is a bad type, which implies that $\mu_{i-1} < 0$, $\sigma^2 b^-_{i-1} \to 2\mu_{i-1}$, and $\sigma^2 b^+_{i-1} \to 0$. Hence:

$$D_i = -2\mu_{i-1} \lim_{\sigma^2 \to 0} \left[\frac{(U - \bar{U}_{i-1}) - (U - \bar{U}_i) e^{b^+_{i-1} y^\star_i}}{e^{b^+_{i-1} y^\star_i} - e^{b^-_{i-1} y^\star_i}} \right] - (U - \bar{U}_i) \lim_{\sigma^2 \to 0} \left[\frac{\sigma^2 b^+_{i-1} e^{b^-_{i-1} y^\star_i}}{e^{b^+_{i-1} y^\star_i} - e^{b^-_{i-1} y^\star_i}} \right]. \quad (89)$$

One possibility (given that $i \leq i^\circ$) is that $\lim_{\sigma^2 \to 0} y^\star_i > 0$. If so, then $e^{b^-_{i-1} y^\star_i} \to 0$ (this is because $b^-_{i-1} \to -\infty$) whereas $e^{b^+_{i-1} y^\star_i}$ remains finite. Given $\sigma^2 b^+_{i-1} \to 0$, the second term above is zero. So:

$$D_i = -2\mu_{i-1} \lim_{\sigma^2 \to 0} \left[\frac{(U - \bar{U}_{i-1}) - (U - \bar{U}_i) e^{b^+_{i-1} y^\star_i}}{e^{b^+_{i-1} y^\star_i}} \right] = -2\mu_{i-1} \left[\frac{(U - \bar{U}_{i-1})}{\lim_{\sigma^2 \to 0} e^{b^+_{i-1} y^\star_i}} - (U - \bar{U}_i) \right]$$

$$\Rightarrow \quad \lim_{\sigma^2 \to 0} y^\star_i = -\frac{\mu_{i-1}}{\rho} \log\left(\frac{U - \bar{U}_{i-1}}{U - \bar{U}_i - \frac{D_i}{2\mu_{i-1}}} \right) = \frac{|\mu_{i-1}|}{\rho} \log\left(\frac{U - \bar{U}_{i-1}}{U - \bar{U}_i + \frac{D_i}{2|\mu_{i-1}|}} \right), \quad (90)$$

where here we used the fact that $b^+_{i-1} \to -\rho/\mu_{i-1}$. This works so long as the logarithm is positive, which requires its argument to exceed one. That is, we require

$$\frac{U - \bar{U}_{i-1}}{U - \bar{U}_i + \frac{D_i}{2|\mu_{i-1}|}} \geq 1 \quad \Leftrightarrow \quad \bar{U}_i - \bar{U}_{i-1} \geq \frac{D_i}{2|\mu_{i-1}|}. \quad (91)$$

This inequality holds for case (c) of claim (i) above. The other (and last remaining) possibility is that $y^\star_i \to 0$. b^+_{i-1} converges, and so $b^+_{i-1} y^\star_i \to 0$ and $e^{b^+_{i-1} y^\star_i} \to 1$. So:

$$D_i = -2\mu_{i-1} \lim_{\sigma^2 \to 0} \left[\frac{\bar{U}_i - \bar{U}_{i-1}}{1 - e^{b^-_{i-1} y^\star_i}} \right] - (U - \bar{U}_i) \lim_{\sigma^2 \to 0} \left[\frac{\sigma^2 b^+_{i-1} e^{b^-_{i-1} y^\star_i}}{1 - e^{b^-_{i-1} y^\star_i}} \right]. \quad (92)$$

Mimicking earlier arguments, $b^-_{i-1} y^\star_i$ cannot diverge or vanish to zero. Given that it converges, and that $\sigma^2 b^+_{i-1} \to 0$, the right-hand term vanishes to zero. Hence:

$$D_i = -2\mu_{i-1} \lim_{\sigma^2 \to 0} \left[\frac{\bar{U}_i - \bar{U}_{i-1}}{1 - e^{b^-_{i-1} y^\star_i}} \right] \quad \Rightarrow \quad \lim_{\sigma^2 \to 0} b^-_{i-1} y^\star_i = \log\left(1 + \frac{2\mu_{i-1}(\bar{U}_i - \bar{U}_{i-1})}{D_i}\right). \quad (93)$$

Of course, $\sigma^2 b_{i-1}^- \to 2\mu_{i-1}$, which yields claim (i) for this case (b). This is valid so long as the argument of the logarithm is strictly positive. This requires

$$1 + 2\mu_{i-1}\frac{\bar{U}_i - \bar{U}_{i-1}}{D_i} > 0 \quad \Leftrightarrow \quad \bar{U}_i - \bar{U}_{i-1} < \frac{D_i}{2|\mu_{i-1}|}, \tag{94}$$

which is the inequality reported in the description of case (b) for claim (i).

Turning to claim (ii), recall that

$$D_{i(-)}(y) \equiv \frac{(U - \bar{U}_i)(b_i^+ e^{-b_i^- y} - b_i^- e^{-b_i^+ y}) - (U - \bar{U}_{i+1})(b_i^+ - b_i^-)}{e^{-b_i^- y} - e^{-b_i^+ y}}. \tag{95}$$

Recall also that as $\sigma^2 \to \infty$, and for $\mu_i < 0$ (which is when $i < i^\circ$), b_i^- diverges to $-\infty$, while b_i^+ converges. Thus, $e^{-b_i^- y} \to \infty$ is the dominant term throughout. That is,

$$\lim_{\sigma^2 \to 0} D_{i(-)}(y) = \lim_{\sigma^2 \to 0} \left[\frac{(U - \bar{U}_i)(b_i^+ e^{-b_i^- y} - b_i^- e^{-b_i^+ y})}{e^{-b_i^- y}} \right] = \lim_{\sigma^2 \to 0} \left[(U - \bar{U}_i) b_i^+ \right] = -\frac{\rho(U - \bar{U}_i)}{\mu_i}, \tag{96}$$

which yields the stated claim. Claims (iii), (iv), and (v) use similar methods.

Finally, consider claim (vi). Suppose that $y/\sigma^2 \to Y$ for some $Y \in (0, \infty)$; hence we are dropping here the $y_i^\star$ notation for simplicity. We have:

$$\sigma^2 D_{i(-)}(y) = \frac{(U - \bar{U}_i)(\sigma^2 b_i^+ e^{-b_i^- y} - \sigma^2 b_i^- e^{-b_i^+ y}) - (U - \bar{U}_{i+1})(\sigma^2 b_i^+ - \sigma^2 b_i^-)}{e^{-b_i^- y} - e^{-b_i^+ y}} \tag{97}$$

$$= \frac{(U - \bar{U}_i)(\sigma^2 b_i^+ e^{-\sigma^2 b_i^- (y/\sigma^2)} - \sigma^2 b_i^- e^{-\sigma^2 b_i^+ (y/\sigma^2)}) - (U - \bar{U}_{i+1})(\sigma^2 b_i^+ - \sigma^2 b_i^-)}{e^{-\sigma^2 b_i^- (y/\sigma^2)} - e^{-\sigma^2 b_i^+ (y/\sigma^2)}} \tag{98}$$

$$\to \frac{(U - \bar{U}_i)(\sigma^2 b_i^+ e^{-\sigma^2 b_i^- Y} - \sigma^2 b_i^- e^{-\sigma^2 b_i^+ Y}) - (U - \bar{U}_{i+1})(\sigma^2 b_i^+ - \sigma^2 b_i^-)}{e^{-\sigma^2 b_i^- Y} - e^{-\sigma^2 b_i^+ Y}}, \tag{99}$$

where the last step takes the limit of (y/σ^2). Now suppose that $i \geq i^\circ$, so that $\mu_i > 0$. This means that $\sigma^2 b_i^+$ has a positive finite limit, while $\sigma^2 b_i^- \to 0$. Hence

$$\sigma^2 D_{i(-)}(y) \to \frac{(U - \bar{U}_i)\sigma^2 b_i^+ - (U - \bar{U}_{i+1})\sigma^2 b_i^+}{1 - e^{-\sigma^2 b_i^+ Y}} = \frac{(\bar{U}_{i+1} - \bar{U}_i)\sigma^2 b_i^+}{1 - e^{-\sigma^2 b_i^+ Y}}$$
$$\to \frac{2\mu_i(\bar{U}_{i+1} - \bar{U}_i)}{1 - e^{-2\mu_i Y}}, \tag{100}$$

which yields the required claim. A similar calculations applies for the case where $i < i^\circ$. □

Proof of Claim (ii) of Proposition 8. We noted that above and below the threshold $x^\star$, the value function is readily solved explicitly. Specifically, the solution eq. (74) applies

$$V_0(x) - U_0 = \frac{(\bar{U} - U_0)(e^{-b_0^- x} - e^{-b_0^+ x})}{e^{-b_0^- x^\star} - e^{-b_0^+ x^\star}} + \frac{(L - U_0)(e^{b_0^+ (x^\star - x)} - e^{b_0^- (x^\star - x)})}{e^{b_0^+ x^\star} - e^{b_0^- x^\star}} \quad \text{and} \tag{101}$$

$$V_1(x) - U_1 = \frac{(W - U_1)(e^{-b_1^- (x - x^\star)} - e^{-b_1^+ (x - x^\star)})}{e^{-b_1^- (\bar{x} - x^\star)} - e^{-b_1^+ (\bar{x} - x^\star)}} + \frac{(\bar{U} - U_1)(e^{b_1^+ (\bar{x} - x)} - e^{b_1^- (\bar{x} - x)})}{e^{b_1^+ (\bar{x} - x^\star)} - e^{b_1^- (\bar{x} - x^\star)}}, \tag{102}$$

where $U_i \equiv u_i/(\rho + \lambda_i)$ for each $i \in \{0, 1\}$ and where the coefficients $b_i^\pm$ satisfy

$$b_i^\pm \equiv \frac{\mu_i \pm \sqrt{\mu_i^2 + 2(\rho + \lambda_i)\xi^2 \sigma_i^2}}{\xi^2 \sigma_i^2}, \tag{103}$$

and where we note here that these coefficients satisfied these properties:

$$\lim_{\xi\to 0}\xi^2 b_1^+ = \frac{2\mu_1}{\sigma_1^2}, \quad \lim_{\xi\to 0} b_1^- = -\frac{\rho+\lambda_1}{\mu_1}, \quad \lim_{\xi\to 0} b_0^+ = -\frac{\rho+\lambda_0}{\mu_0}, \quad \text{and} \quad \lim_{\xi\to 0}\xi^2 b_0^- = \frac{2\mu_0}{\sigma_0^2}, \tag{104}$$

and where we note that b_1^+ and b_0^- diverge to $\pm\infty$ whereas b_1^- and b_0^+ both converge.

We now exploit once more the smoothness of the value function. Equations (29) and (30) become

$$V_1'(x^\star) = \frac{(U_1-\bar{U})(b_1^+ e^{-b_1^-(\bar{x}-x^\star)} - b_1^- e^{-b_1^+(\bar{x}-x^\star)}) - (U_1-W)(b_1^+ - b_1^-)}{e^{-b_1^-(\bar{x}-x^\star)} - e^{-b_1^+(\bar{x}-x^\star)}} \quad \text{and} \tag{105}$$

$$V_0'(x^\star) = \frac{(U_0-L)(b_0^+ - b_0^-) - (U_0-\bar{U})(b_0^+ e^{b_0^- x^\star} - b_0^- e^{b_0^+ x^\star})}{e^{b_0^+ x^\star} - e^{b_0^- x^\star}}. \tag{106}$$

Multiplying by the noise-scaling parameter ξ^2 and taking the limit as $\xi^2 \to 0$ yields

$$\xi^2 V_1'(x^\star) \to \frac{2\mu_1\left[(U_1-\bar{U}) - (U_1-W)e^{-(\rho+\lambda_1)(\bar{x}-x^\star)/\mu_1}\right]}{\sigma_1^2} \quad \text{and} \tag{107}$$

$$\xi^2 V_0'(x^\star) \to \frac{2|\mu_0|\left[(U_0-L)e^{-(\rho+\lambda_0)x^\star/|\mu_0|} - (U_0-\bar{U})\right]}{\sigma_0^2}. \tag{108}$$

We write $x^\diamond = \lim_{\xi^2\to 0}$ for the threshold that equates these two limiting expressions. These satisfies

$$\frac{|\mu_0|}{\sigma_0^2}\left[(U_0-L)e^{-(\rho+\lambda_0)x^\diamond/|\mu_0|} - (U_0-\bar{U})\right] = \frac{\mu_1}{\sigma_1^2}\left[(U_1-\bar{U}) + (W-U_1)e^{-(\rho+\lambda_1)(\bar{x}-x^\diamond)/\mu_1}\right]. \tag{109}$$

The left-hand side is decreasing in $x^\diamond$, while the right-hand side is increasing, and so this (of course, given the uniqueness of the equilibrium) has a unique solution. This solution $x^\diamond$ falls (which means that a lower threshold is needed to induce high effort, so favoring the success of the team) in response to anything that lowers the left-hand side or raises the right-hand side. The comparative-static claim regarding the relative volatility in the two regimes follows straightforwardly. □